Gravity vs matter: Stars as laboratory to test theories of gravity

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Motivation. What is done so far

So far, the modified gravity community is used to play mainly in the cosmological framework and/or astrophysical ones:

- neutron stars[1]
- black holes[2]
- exotic environment (wormholes, branes, cosmic strings, ...).

Growing data sets, especially on gravitational waves detections, allows to test and rule out[3] some of the gravitational theory proposals.

Regarding stars in modified gravity[4]

- M-R relations (degeneracy with EoS's), stability problem, I-Love-Q relations
- Issues: matching conditions (ill behaviour of surface quantities), mass problem, GR scenarios not working (Birkoff theorem, collapse model), non-physical matter
- Non-relativistic stars: energy production (light elements burning), evolutionary scenarios, mass' limits, crystallization and convective processes, age of astrophysical objects, constraining theories (true stars vs brown dwarfs, Hayashi tracks, helioseismology,...)

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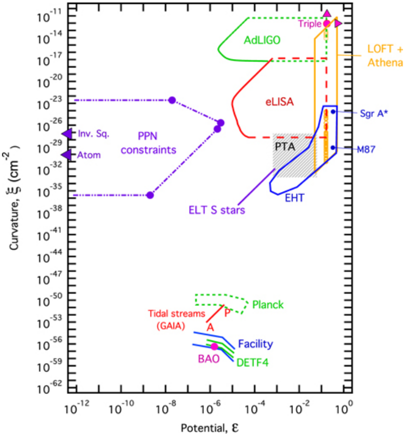
[1] B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 119 (2017)161101;

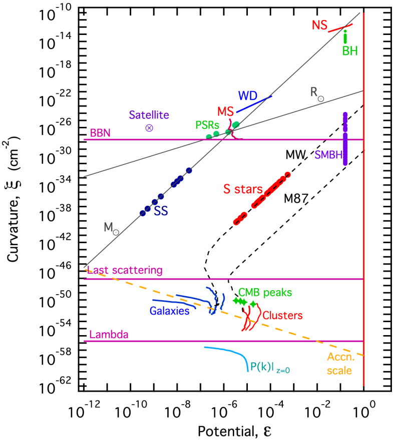
[2] B. P. Abbottet al.[LIGO Scientific and Virgo Collabo-rations], Phys. Rev. Lett.116(2016) 061102; K. Akiyamaet al.[Event Horizon Telescope Collabora-tion], Astrophys. J.875(2019) L1

[3] J. Ezquiaga et al., Phys. Rev. Lett. 119, 251304 (2017); S. Boran et al., Phys. Rev. D 97, 041501 (2018); R. Sanders, Int.J.Mod.Phys.D, 27, 14, (2018) 1847027; B. P. Abbottet al.[LIGO Scientific and Virgo Collabo-rations], Phys. Rev. Lett. 123, 011102 (2019); S. Jana et al., Phys. Rev. D 99, 044056 (2019);

[4] G.J. Olmo, D. Rubiera-Gracia, **AW**, Physics Reports 876 (2020)

The stellar and galaxy curvature regime not considered too much in MG...





Untested regime in the **the galaxy and stellar physics regime**. It could potentially hide the onset of corrections to GR (T. Baker et al 2015 ApJ 802, 63).

Curvature $\xi = (R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta})^{\frac{1}{2}} = \sqrt{48}\frac{GM}{r^3c^2}$

Potential $\varepsilon = \frac{GM}{rc^2}$

Further motivation: tests of gravity

- Helioseismology[5]
- Low-mass stars[6]: Hydrogen burning, Hayashi tracks, Lithium burning
- II population stars[7]
- Brown dwarfs (cooling process)[8]
- Giant exoplanets (formation and cooling process)[9]
- White dwarfs (cooling process, crystallization)[10]
- Earthquakes and marsquakes[11]; + neutrino telescopes (neutrino tomography)[12]

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5 I. Saltas, I. Lopes, PRL 123 (2019) 9, 091103; I. Saltas, J. Christensen-Dalsgaard, arXiv:2205.14134

6 J. Sakstein, PRL 115 (2015) 201101; **AW**, PRD 102 (2020) 12, 124045; **AW**, PRD 103 (2021) 4, 044037

7 S. Chowdhury, T. Sarkar, JCAP 05 (2021) 040

8 M. Benito, **AW**, PRD 103 (2021) 6, 064032; A. Kozak, K. Soieva, **AW**, arXiv:2205.12812.

9 **AW**, PRD 104 (2021) 10, 104058; **AW**, PRD 105 (2022) 124053

10 S. Kalita, L. Sarmah, **AW**, arXiv:2209.02095; in preparation

11 A. Kozak, **AW**, PRD 104 (2021) 8, 084097; IJGMP 19 (2022) Supp01, 2250157; Universe 8 (2021) 1, 3; in preparation

12 A. Donini, S. Palomares-Ruiz, J. Salvado, Nature Physics 15.1 (2019): 37-40

Basic equations of stellar (sub-)structure[13]

(nonsphericity, magnetic fiels, time-dependency,... ignored here

The structural and thermal differential relations

$$\frac{dp}{dr} = -\frac{Gm}{r^2}\rho, \quad \frac{dm}{dr} = 4\pi r^2\rho, \quad \frac{dL}{dr} = 4\pi r^2\epsilon\rho$$

Criteria for the heat transfer model (e.g. Schwarzschild criterion)

$\nabla_{\text{rad}} \leq \nabla_{\text{ad}}$ pure diffusive radiative transport or conduction

$\nabla_{\text{rad}} > \nabla_{\text{ad}}$ adiabatic convection is present locally

where $\nabla_{\text{rad}} := \left(\frac{d\ln T}{d\ln p}\right)_{\text{rad}} = \frac{3\kappa_{rc}Lp}{16\pi acGmT^4}$

Microscopic physics (X is shorthand for composition)

$$p = p(\rho, T, X), \quad E = E(\rho, T, X), \quad \kappa = \kappa(\rho, T, X), \quad \epsilon = \epsilon(\rho, T, X)$$

Boundary conditions at the center ($m = 0$, $r = L = 0$) and at the surface ($m = M$, $\rho = T = 0$).

[13] see e.g. *Stellar Interiors, Physical Principles, Structure and Evolution*, C.J. Hansen, S.D. Kawaler, V. Trimble

Gravity vs matter: motivation based on a number of indications

- Effective quantities: opacity[14], ...
- Modifications introduced by MG to pressure[15]
- Chemical reactions rates depend on gravity[16]
- Specific heat and crystallization depend on MG[17]
- Chemical potential depends on gravity[18]
- Elementary particle interactions modified by MG (dependence of the metric on the local energy-momentum distributions[19]
- EoS depends on relativistic effects introduced by GR[20]
- Thermonuclear processes...?[21]
- Fermi equation of state depends on (modified) gravity[22]

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14 J. Sakstein, PRD 92 (2015) 124045; ...

15 H-Ch. Kim, PRD 89 (2014) 064001

16 P. Lecca, J. Phys.: Conf. Ser. 2090 (2021) 012034

17 S. Kalita, L. Sarmah, **AW**, in preparation

18 I.K. Kulikov, P.I. Pronin, Int. J. Theor. Phys. 34, (1995) 9

19 A.D.I Latorre, G.J. Olmo, M. Ronco, PRB 780, 294 (2018)

20 G.M. Hossain, S. Mandal, JCAP 02 (2021) 026; PRD 104 (2021) 123005

21 J. Sakstein, PRD 92 (2015) 124045, **AW** PRD 103 (2021) 4, 044037; M. Guerrero, **AW**, in preparation

22 **AW**, arXiv:2208.04023

Hot topics for Aneta

- Matter versus gravity (equation of state in modified gravity, lithium problem, nucleosynthesis, energy production)
- Brown dwarf stars and exoplanets suitable to test and to constrain theories of gravity
- Stellar and substellar evolution differs with respect to various theories of gravity
    - formation of jovian planets and solar systems, habitability of exoplanets
    - stages of stellar evolution, age, substellar and WD's cooling
- Seismology versus gravity
    - earthquakes and marsquakes to test modified gravity
    - helioseismology
- Crystallization process in white dwarfs
- Lensing of GW's by astrophysical objects
- Scalarization and screening mechanism breaking in compact and stellar objects
- ...

Palatini gravity in a nutshell

$$S = S_{\rm g} + S_{\rm m} = \frac{1}{2\kappa}\int \sqrt{-g} f(\hat{R}) d^4x + S_{\rm m}(g_{\mu\nu}, \psi_m),$$

where $\hat{R} = \hat{R}^{\mu\nu}(\hat{\Gamma})g_{\mu\nu}$. Modified field equations wrt $g_{\mu\nu}$ and $\hat{\Gamma}$ are

$$f'(\hat{R})\hat{R}_{\mu\nu} - \frac{1}{2}f(\hat{R})g_{\mu\nu} = \kappa T_{\mu\nu}, \quad \text{GR:} \quad R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\hat{\nabla}_\beta(\sqrt{-g} f'(\hat{R})g^{\mu\nu}) = 0 \quad \rightarrow \quad h_{\mu\nu} = f'(\hat{R})g_{\mu\nu}.$$

The trace of the first MFE wrt $g_{\mu\nu}$ gives the structural equation

$$f'(\hat{R})\hat{R} - 2f(\hat{R}) = \kappa\mathcal{T},$$

where $\mathcal{T}$ is a trace of e-m tensor $T_{\mu\nu}$ wrt $g_{\mu\nu}$, provides $\hat{R} = \hat{R}(\mathcal{T})$.

- Non-linear system of a second order PDE.
- $f(\hat{R}) = \hat{R} - 2\Lambda$ is fully equivalent to the Einstein $R - 2\Lambda$.
- Any $f(\hat{R})$ vacuum solution $\rightarrow$ Einstein vacuum solution with the cosmological constant.
- Modifies non- and relativistic stellar structure equations[23].

[23] K. Kainulainen et al, PRD. 76 (2007) 043503; **AW**, EPJC 78 (2018) 421; **AW** EPJC 79 (2019) 51; A. Sergyeyev, **AW**, EPJC 80

Non-relativistic equations of Palatini $f(\hat{R})$ gravity

For an analytic function $f(\hat{R}) = \sum_{i=0} \bar{\alpha}_i \hat{R}^i$ the Poisson equation[24]

$$\nabla^2\Phi \approx \frac{\kappa}{2}(\rho + 2\bar{\alpha}\nabla^2\rho)$$

where α comes from the quadratic term of the Lagrangian (we neglect cosmological constant)

$$f(\hat{R}) = \hat{R} + \beta\hat{R}^2 \tag{1}$$

For spherical-symmetric spacetime the gravitational potential[25]

$$\Phi(r) = -\frac{GM}{r} - 4\pi G\int_r^R \Big(\rho(r)r - 2\alpha\rho'(r)\Big)dr.$$

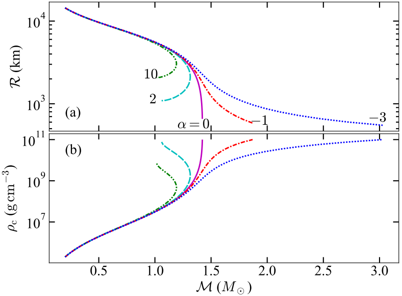
The hydrostatic equilibrium equation

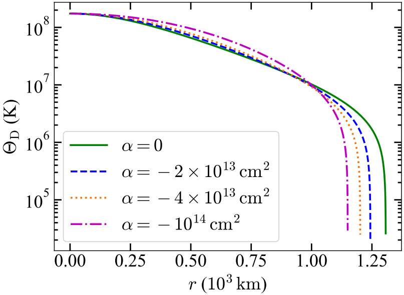
$$\frac{d\Phi}{dr} = -\rho^{-1}\frac{dP}{dr},$$

[24] J. D. Toniato, D. C. Rodrigues, **AW**, PRD 101 (2020) no. 6, 064050; A. Hernandez-Arboleda, D. C. Rodrigues, **AW** arXiv:2204.03762

[25] **AW**, arXiv:2208.04023

White dwarfs and modified gravity[26]





Left panel: with the use of the Chandrasekhar equation of state. Right panel: Variation of Debye temperature inside a modified gravity inspired carbon WD with different α for $\rho_c = 10^{10}\,\mathrm{g\,cm^{-3}}$.

Debye temperature: $\Theta_D = 0.174 \times 10^4 \frac{2Z}{A}\sqrt{\rho}$

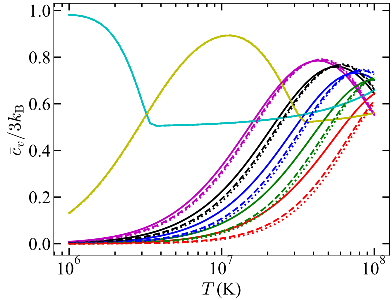
Mean specific heat: $\bar{c}_v = \frac{1}{\mathcal{M}} \int_0^{\mathcal{M}} \left[\frac{3}{2} \frac{k_B \pi^2}{3} Z \frac{k_B T}{\epsilon_F} + 9k_B \left(\frac{T}{\Theta_D} \right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx \right] dm$

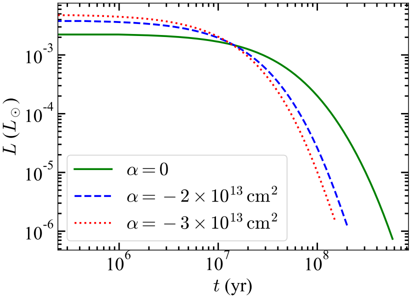
Cooling equation: $L = \frac{3k_B \mathcal{M}}{A m_H} \left(-\frac{\bar{c}_v}{3k_B} + \rho_s q \frac{1}{\mathcal{M}} \frac{dm}{dr} \frac{dr}{d\rho} \right) \frac{dT}{dt}$

[26] S. Kalita, L. Sarmah, **AW**, PRD 105 (2022) 2, 024028; arXiv:2209.02095; in preparation

White dwarfs' cooling[27]

Cooling equation: $$L = \frac{3k_B \mathcal{M}}{A m_H}\left(-\frac{\bar{c}_v}{3k_B} + \rho_s q \frac{1}{\mathcal{M}}\frac{dm}{dr}\frac{dr}{d\rho}\right)\frac{dT}{dt}$$



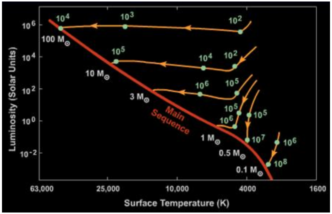


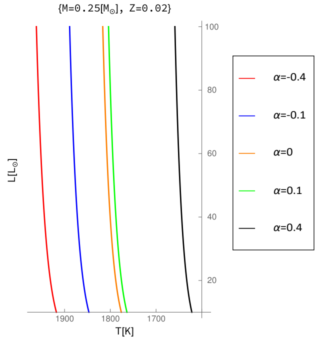
Left: $\bar{c}_v$ as a function of T for different carbon WDs; $\rho_c \in [10^5 - 10^{11}]$ g cm^{-3} for $\alpha = 0$ (solid lines), $\alpha = -2\times10^{13}\,\mathrm{cm}^2$, and $\alpha = -3\times10^{13}\,\mathrm{cm}^2$.

Right: Luminosity as a function of time for carbon WDs with $\rho_c = 10^{11}\,\mathrm{g\,cm}^{-3}$. The initial luminosity corresponds to a surface temperature of $\approx 10^7$ K.

[27] S. Kalita, L. Sarmah, **AW**, in preparation

Early stellar evolution: PMS low-mass stars on Hayashi tracks [28]





Simplified photosphere's and opacity model

Early evolutionary (Hayashi) track

$$T_{\text{ph}} = 2487.77\mu^{\frac{13}{51}} \left(\frac{L}{L_\odot}\right)^{\frac{1}{102}} \times \left(\frac{M}{M_\odot}\right)^{\frac{7}{51}} \left(\frac{\left(\frac{1-\frac{4\alpha}{3\delta}}{Z}\right)^{\frac{4}{3}}}{\xi_R^5\sqrt{-\theta'}} \right)^{\frac{1}{17}} \text{K}$$

Convective/radiative processes ruled by

Schwarzschild criterion

$\nabla_{rad} \leq \nabla_{ad}$ pure diffusive radiative or conductive transport

$\nabla_{rad} > \nabla_{ad}$ adiabatic convection is present locally.

$$\nabla_{rad} = \frac{3\kappa_{rc} L p}{16\pi a c G m T^4}\left(1 - \frac{4\alpha}{3\delta}\right)^{-1}$$

[28] **AW**, PRD 102 (2020) 124045

Lithium abundance is a gravitational model dependent quantity [29]

Lithium depletion boundary method - the most reliable method for young globular clusters' age determination; the lithium test used in order to distinguish brown dwarfs from true stars

It is used to calibrate other age determination methods!

The depletion rate in a star with mass M and hydrogen fraction X:

$$M\frac{\mathrm{d}f}{\mathrm{d}t} = -\frac{Xf}{m_H}\int_0^M \rho\langle\sigma v\rangle dM,$$

where the non-resonant reaction rate for the temperature range $T < 6\times 10^6$K is given by

$$N_A\langle\sigma v\rangle = Sf_{\mathrm{scr}}T_{c6}^{-2/3}\exp\left[-aT_{c6}^{-\frac{1}{3}}\right]\ \frac{\mathrm{cm}^3}{\mathrm{s\ g}},$$

where $T_{c6}\equiv T_c/10^6$K; the parameters S, a are fitted to the reaction rate $^7\mathrm{Li}(p,\alpha)\,^4\mathrm{He}$; f_{scr} is the screening correction factor.

[29] **AW**, PRD 103 (2021) 4, 04403

Lithium abundance is a gravitational model dependent quantity[30]

The lithium depletion $\mathcal{F}$ in Palatini gravity for $T < 6 \times 10^6 K$ is ($u = aT_{c6}^{-1/3}(t)$)

$$\mathcal{F} \equiv \ln \frac{f_0}{f} = 1.15 \times 10^{13}\ T_{3\mathrm{eff}}^{-4} \left(\frac{X}{0.7}\right) \left(\frac{0.6}{\mu_{\mathrm{eff}}}\right)^6 \left(\frac{M_\odot}{M}\right)^3 \times S f_{\mathrm{scr}} a^{16} g(u) \frac{\xi_R^4 (-\theta'(\xi_R))^2 \Omega}{\delta^2}$$

α	$T_c/10^6$K	t [Myr]	$R/R_\odot$	$L/L_\odot$
-0.4	3.48	3.21	1.85	25.3×10^{-2}
-0.1	3.18	7.48	1.28	14.4×10^{-2}
-0.001	3.129	7.76	1.19	14.1×10^{-2}
0 (GR)	2.98	12.42	1.03	10.3×10^{-2}
0.001	3.128	7.78	1.19	14×10^{-2}
0.1	3.098	7.25	1.13	14.7×10^{-2}
0.4	3.093	3.57	1.06	23.6×10^{-2}

- much shorter contraction and ^{7}Li burning phase
- can explain the age of WD KIC 8145411
- noticeable effect on the total stars' luminosity which contributes to the galaxy brightness
- can contribute to cosmological lithium problem[a]

[a] A. Kozak, M. Saal, **AW**, in progress

[30] **AW**, PRD 103 (2021) 4, 04403; D. Gomes, **AW**, arXiv:2206.04464

Hydrogen burning - minimal main sequence mass[33]

... is the lowest mass where the rate-limiting reaction for hydrogen burning can be sustained stably.
... "it is mass that a pre-main-sequence star must have in order to jump into the Main Sequence family"

It means that energy generated in the core is compensated by energy radiated from the surface, which corresponds to the mass where

$$\boxed{L_{\text{hydrogen burning}} = L_{\text{photosphere}}}$$

We want to use this fact for constraining modified gravity theories

The observational bound[31]: M-dwarf star G1 866C with the mass $(0.0930 \pm 0.0008)M_\odot$

The GR theoretical prediction[32]: $\sim 0.08 - 0.09 M_\odot$

[31] D. Segransan et al., Astron. Astrophys. 364 (2000) 665

[32] A. Burrows, J. Liebert, Rev. Mod. Phys. 65, 301 (1993)

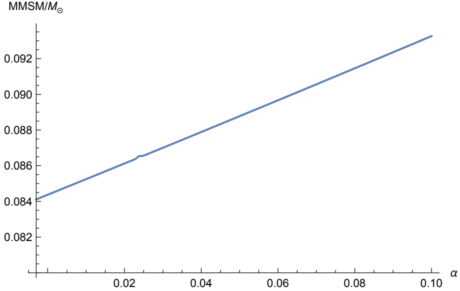
[33] J. Sakstein, PRL 115 (2015) 201101; G. Olmo, D. Rubiera-Garcia, **AW**, PRD 100 (2019) 4, 044020

Testing and constraining the theory with MMSM [34]

$$M_{-1}^{MMSM} = 0.290 \frac{\gamma_{3/2}^{1.32}\omega_{3/2}^{0.09}}{\delta_{3/2}^{0.51}} \frac{(\alpha_d + \eta)^{1.509}}{\eta^{1.325}} \left(1 - 1.31\alpha \frac{\left(\frac{\alpha_d+\eta}{\eta}\right)^4}{\delta_{3/2}\kappa_{-2}}\right)^{0.111}$$

The observational bound: M-dwarf star G1 866C with the mass $(0.0930 \pm 0.0008)M_\odot$

The GR theoretical prediction: $\sim 0.08 - 0.09 M_\odot$

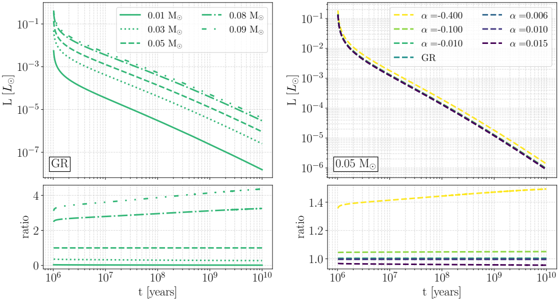


One needs to take into account the realistic **electron degeneracy & opacity** description!

[34] G. Olmo, D. Rubiera-Garcia, **AW**, PRD 100, 4, 044020 (2019); M. Guerrero, D. Rubiera-Garcia, **AW** EPJC 82 (2022) 8

Brown dwarfs as laboratories to test gravity[36] and dark matter models[37]

- more realistic description of the partially-degenerate state that characterizes brown dwarfs and low-mass stars wrt the previous studies [35]
- the hydrogen metallic-molecular phase transition between the interior of the brown dwarf and its photosphere taken into account
- luminosity and degeneracy as functions of time (affected by MG)



[35] G.J. Olmo, D. Rubiera-Garcia, **AW**, Phys. Rev. D 100 (2019) 4, 044020

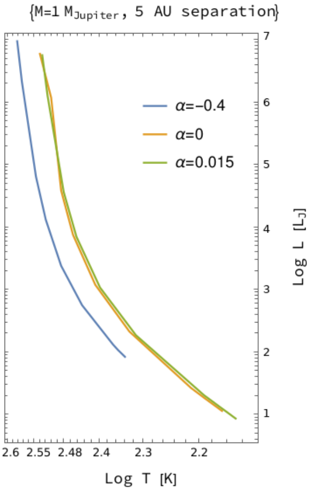
[36] M. Benito, **AW**, PRD 103 (2021) 6, 064032; A. Kozak, K. Soieva, **AW**, arXiv:2205.12812

[37] R.K. Leane, J. Smirnov, PRL, 126(16), 161101 (2021).

Jupiter and jovian (exo-)planets[38]

The effective temperature, luminosity (in Jupiter's luminosity), and age ratio with respect to the GR values at the given point of the jovian planet's evolution for the given radius R for $\alpha = -0.4$.

R (10^9m)	T_{eff} (K)	L/L_J	t_α / t_{GR}
10	381	9×10^6	1.5
5	372	2×10^6	1.7
1	346	0.6×10^5	1.6
0.5	329	13×10^3	1.7
0.25	304	2×10^3	1.9
0.15	273	5.6×10^2	2.4
0.1	233	1.3×10^2	3.8
0.095	227	1.1×10^2	4.3
0.09	219	83	4.7



- Effective temperature and luminosity differ in modified gravity
- A jovian planet can be younger/older than predicted by GR
- Fragmentation process and opacity mass limit also impacted by Palatini gravity
- Accretion model depends on modified gravity

38 **AW** PRD 104 (2021) 10, 104058

Accretion model of the jovian planets' formation[39]

- Isolation mass (dashed line) and the minimum planet's core mass needed to sustain a massive envelope given by

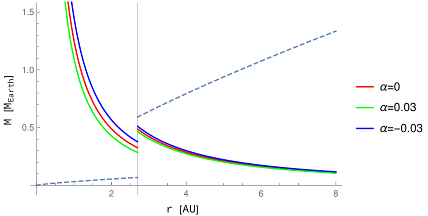
$$M_c > \left(\frac{3}{32\pi}\right)^{\frac{1}{3}} \left(\frac{h}{r}\right)^3 \frac{\left(M_* \left(1 + 3\alpha \frac{M_*}{\pi r^3}\right)\right)^{\frac{3}{2}}}{\rho_m^{\frac{1}{3}} r^{\frac{3}{2}} \left(1 + 4\alpha\rho_m\right)^{\frac{3}{2}}}$$

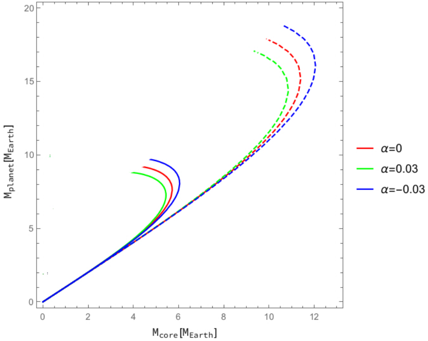
  The snowline at $r = 2.7$ AU and other properties of the protoplanetary disk with a Solar mass star are given assuming the Hayashi minimum mass Solar Nebula model. For the interior to the snowline, $\rho_m = 3$ g cm^{-2}, and beyond the snowline, $\rho_m = 1$ g cm^{-2}.

- The planet's core mass $M_{\rm core}$ dependence on the total mass M_p given by

$$M_{\rm core} = M_p - \frac{C_1}{\kappa_R \dot{M}_{\rm core}} \frac{M_p^4}{M_{\rm core}^{2/3}} \left[\ln \frac{r_{\rm out}}{R_s} + \alpha C_2 M_p\right]$$

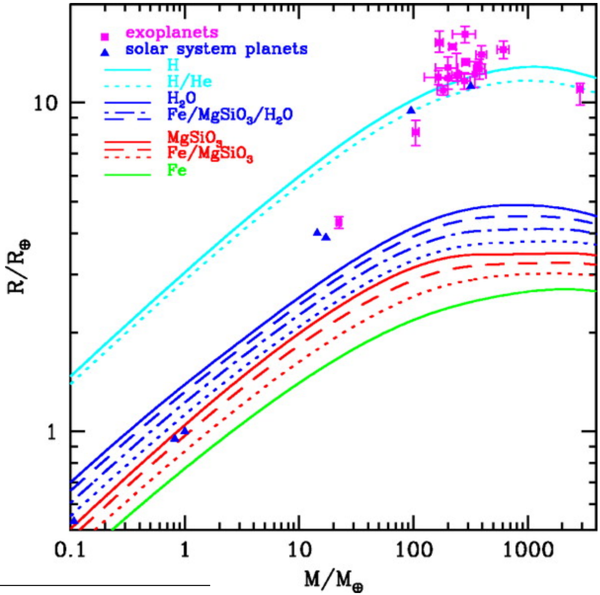
  for a constant opacity and two constant core accretion rates. The dashed curves are given by the five-fold rate with respect to the solid ones.





39 D. J. Stevenson, Planetary and Space Science 30.8 (1982), 755-764; **AW**, PRD 105 (2022) 12, 124053

Terrestrial (exo-)planets in Newtonian gravity[40]



[40] S. Seager, et al., The Astrophysical Journal 669.2 (2007): 1279

Terrestrial (exo-)planets in Palatini gravity[41]

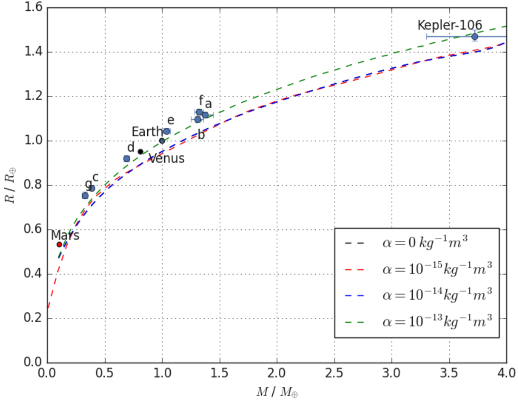
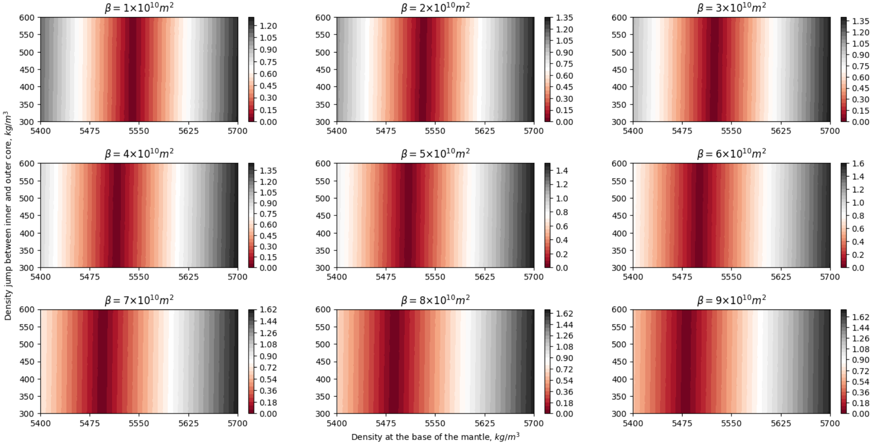


Figure: Mass-radius relation for small planets composed of two layers: iron core and perovskite silicate mantle. The results were obtained for different values of the Starobinsky parameter $\alpha = 2c^2\kappa^2\beta$. The solar-system planets were included, as well as some TRAPPIST-1 exoplanets, denoted by letters.

[41] A. Kozak, **AW**, IJGMMP 19 (2022) Supp01, 2250157; E. Agol et al., Planet. Sci. 2(2021) 1.

Terrestrial planets - seismology vs gravity I [43]

Constraining theory (moment of inertia $I = 8.01736 \pm 0.00097 \times 10^{37}$ kg m^2 and mass $M = 5.9722 \pm 0.0006 \times 10^{24}$ kg)



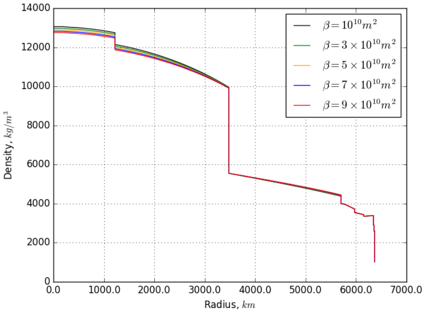
Information on matter density inside the planet, and on characteristics and abundances of light elements in the outer core[42]

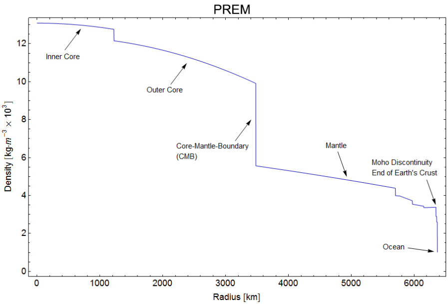
[42] A. Donini, S. Palomares-Ruiz, J. Salvado, Neutrino tomography of Earth. Nature Phys 15, 37-40 (2019)

[43] A. Kozak, **AW**, in preparation

Terrestrial planets - seismology vs gravity II [45]

The Earth's density profile (inner and outer core, mantle + outer layers given by the Birch law)





On the RHS: Palatini gravity ($\Delta\rho = 600$, $\rho_m = 5550$); on the left: Preliminary Reference Earth Model (PREM) A. M. Dziewonski, D. L. Anderson, Phys. Earth Plan. Int. 25 (1981) 297.

Exoplanets properties: central values and CMB are affected by modified gravity[44]

[44] A. Kozak, **AW**, Universe 8 (2021) 1, 3

[45] A. Kozak, **AW**, PRD 104 (2021) 8, 084097; IJGMMP 19 (2022) Supp01, 2250157; in preparation

Summary and conclusions

- Tests of gravity with the use of stars and substellar objects (BD, (exo)-planets, seismology)
- We must be consistent in describing physical systems in different scales
- We should consider more realistic models on both sides: gravity and matter - rotating bodies, magnetic fields, ..., opacities (atmosphere)
- More research on matter properties in the MG framework is necessary

Thanks!

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