Application of the Dual Rotating Retarder Polarimeter Technique to the Calibration of a Liquid Crystal Variable Retarder

Luca Zangrilli, Gerardo Capobianco, Chiara Buscemi,
Filippo Crudelini, Silvano Fineschi

INAF - Osservatorio Astronomico di Torino

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PREPARED BY
Luca Zangrilli INAF – Osservatorio astronomico di Torino.

1. Introduction

We describe a preliminary application of a Dual Rotating Retarder Mueller Spectro-Polarimeter, measuring the retardance and the fast axis orientation of a Liquid Crystal Variable Retarder (LCVR), which was originally mounted in the K-Corona polarimeter EKPol, used during the 29th March 2009 total solar eclipse (Zangrilli et al. 2006). The study proceeded in two steps:

1) Laboratory assembly and test with known polarimetric samples.
2) Measurement of the basic polarimetric properties of a LCVR, i.e. retardance and fast axis orientation.

The results of this activity will be compared with previous laboratory calibrations in a forthcoming report. The same laboratory setup will be used for the calibration of the EKPol polarimeter fully assembled, and to implement a laboratory facility for extensive tests of LCVR samples.

The Dual Rotating Retarder Polarimeter (DRRP) technique has been originally proposed by Azzam (1978), as an alternative to the classic simple polarimetric calibration technique. The DRRP methodology is based on the encoding of the polarimetric properties of a sample, described by its Mueller matrix elements (see Appendix B), in a signal modulated by two retarders (quarter waves in the present approach), placed before and after the sample, and rotating in a 1:5 ratio (Figure 1). The advantage of this method is the possibility of a complete measure of the Mueller matrix of the sample, allowing to evaluate deviations from the ideality, such as diattenuation and depolarization. The analysis formalism has been developed by Azzam (1978), and with a major emphasis on the setup systematic errors by Goldestein & Chipman (1990) and Chenault et al. (1992). The algorithms for the data analysis are reported in Appendix C.

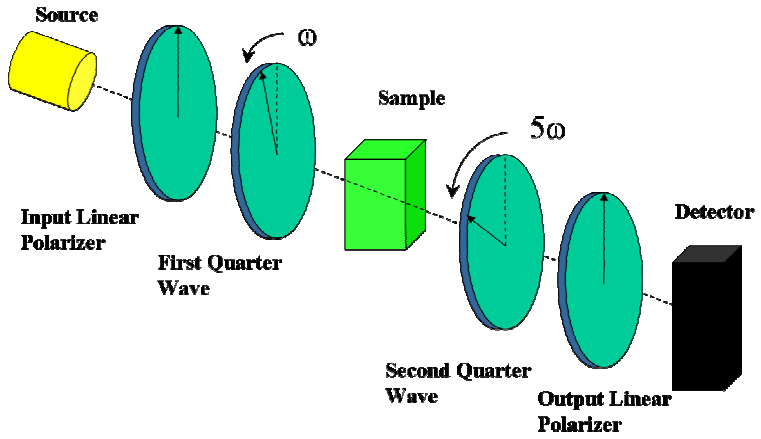


Figure 1: Dual Rotating Retarder Scheme

2. Experimental Setup

2.1 Setup Description

The laboratory setup consists of an halogen light source and a monochromator, selecting the wavelength of the output radiation in a band of about 23[A], which goes into a box ensuring the light tightness and containing the polarimetric components of the setup and those to be tested. Inside this box a linear polarizer defines the polarization axis of the input radiation. Owing to the low levels from the light source, a PMT has been used as a detector. The stability of the source has been monitored by a second detection channel, in which a photodiode is fed by a beam splitter put after the input linear polarizer. The sample is mounted between two rotating quarter waves, modulating the signal, which is then analyzed by a second linear polarizer and recorded by the PMT (see Figures 2, 3, 4, 5 and 6). This measurement scheme is also referred to as Mueller Spectro-Polarimeter (MSP), owing to the fact that the sample properties are also studied as a function of the wavelength.

List of setup components

Polarimetric components

- Input Linear Polarizer: UVCS n° 3
- First Quarter Wave: Meadowlark H3051
- Second Quarter Wave: Meadowlark H3050
- Output Linear Polarizer: UVCS n° 1

Mechanical Rotators

Rotation Stage PI model M-038.DG

- QW1: Serial number 1379
- QW2: Serial number 1378

Controllers

- Precision Motor Controller PI model C-84420, serial 5000111
- D2040 Meadowlark

Detectors

- PMT Hamamatsu, model H7155-21
  - main channel: n° 54690010
  - secondary control channel: n° 56320009

Figure 2: Optical bench and setup instrumentation

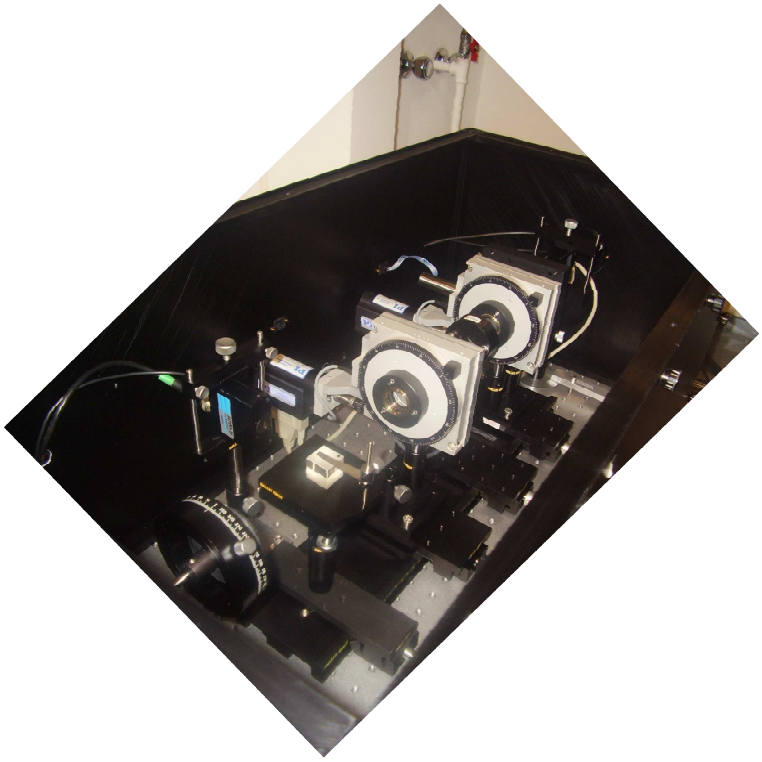
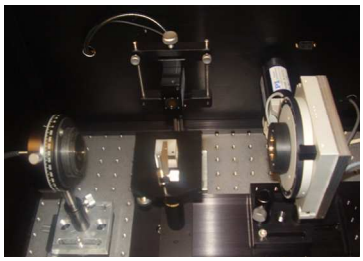
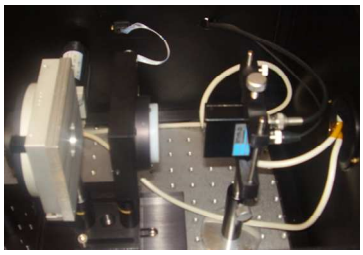


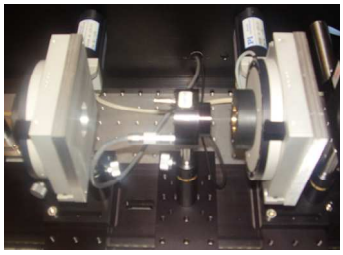
Figure 2: Comprehensive view of the polarimeter assembly



a) Canale di Controllo



b) Canale di Misura



c) Campione: LCPR

Figure 3: Details of different polarimeter sections

M-038 Rotation Stage

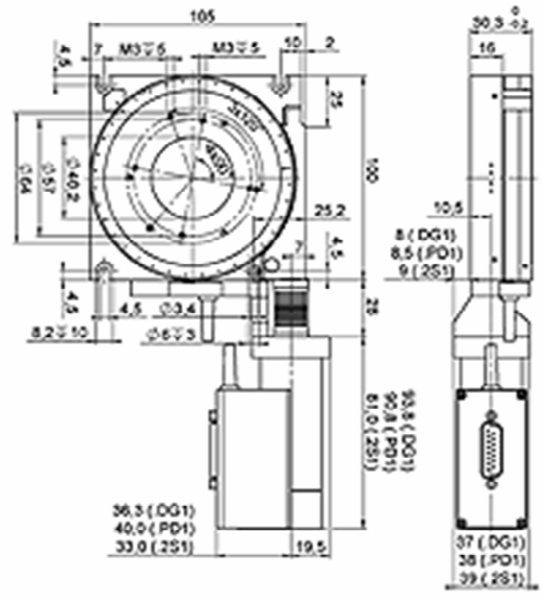


Figure 4: M-038.DG1 Rotation Stages with Worm Gear Drive (from PI Catalogue)

Technical Data

- Model: M-038.DG1 Rotation Stages with Worm Gear Drive
- Ultra-High Resolution
- Max. Velocity 90°/s
- Continuous Rotation Range
- Preloaded Worm Drive for Zero Backlash
- ActiveDrive™ Manual, DC-Servo and Stepper-Motor Drives
- Clear Aperture ∅ 40.2 mm
- Vacuum-Compatible Versions Available to 10^{-6} hPa

Motion and positioning

Active axes:	Rotation
Rotation range:	>360°
Integrated sensor:	Rotary encoder
Sensor resolution:	2000 steps/rev.
Design resolution:	0.60 (35 x 10^{-6}) µrad (*)
Min. incremental motion:	3.5 µrad
Backlash:	200 µrad
Unidirectional repeatability:	20 µrad
Wobble:	<75 µrad
Max. velocity:	6°/s

Mechanical properties

Worm gear ratio:	176:1
Gear ratio:	2401:81 ≈ 29.6:1
Motor resolution	- steps/rev.
Max. load/axial force:	±400N
Maximum torque (θ_X, θ_Y):	±6
Maximum torque CW**	2 Nm
Maximum torque CCW**	0.8Nm

Drive properties

Motor type:	DC Motor, gearhead
Electrical power:	3W
Reference switch :	Hall-effect

Miscellaneous

Operating voltage:	12 V differential
Operating temperature range:	-20 to +65°C
Material :	Aluminum
Mass:	1.25Kg

* 2-phase stepper motor, 24 V chopper voltage, max. 0.8 A/phase, 400 full steps/rev., motor resolution with C-663 stepper motor controller
** CW: clockwise; CCW: counter-clockwise

Rotator Controller

Model: C-844 Two and Four Channel Precision DC Motor Controllers

Figure 5: C-844 Two and Four Channel Precision DC Motor Controllers (from PI Catalogue)

2.2 MSP Control, Data Acquisition and Data Analysis Software

The Software for the MSP control, data acquisition and data analysis is written in LabVIEW[1] language, which is designed for the control of laboratory instrumentation. This software, called SpektroSoft, allows user to choose different operation modes for the MSP or to perform the frequency analysis of the acquired modulation curves. Screenshot is reported in the following Figure 7.

[1] LabVIEW is a development tool registered by National Instruments.

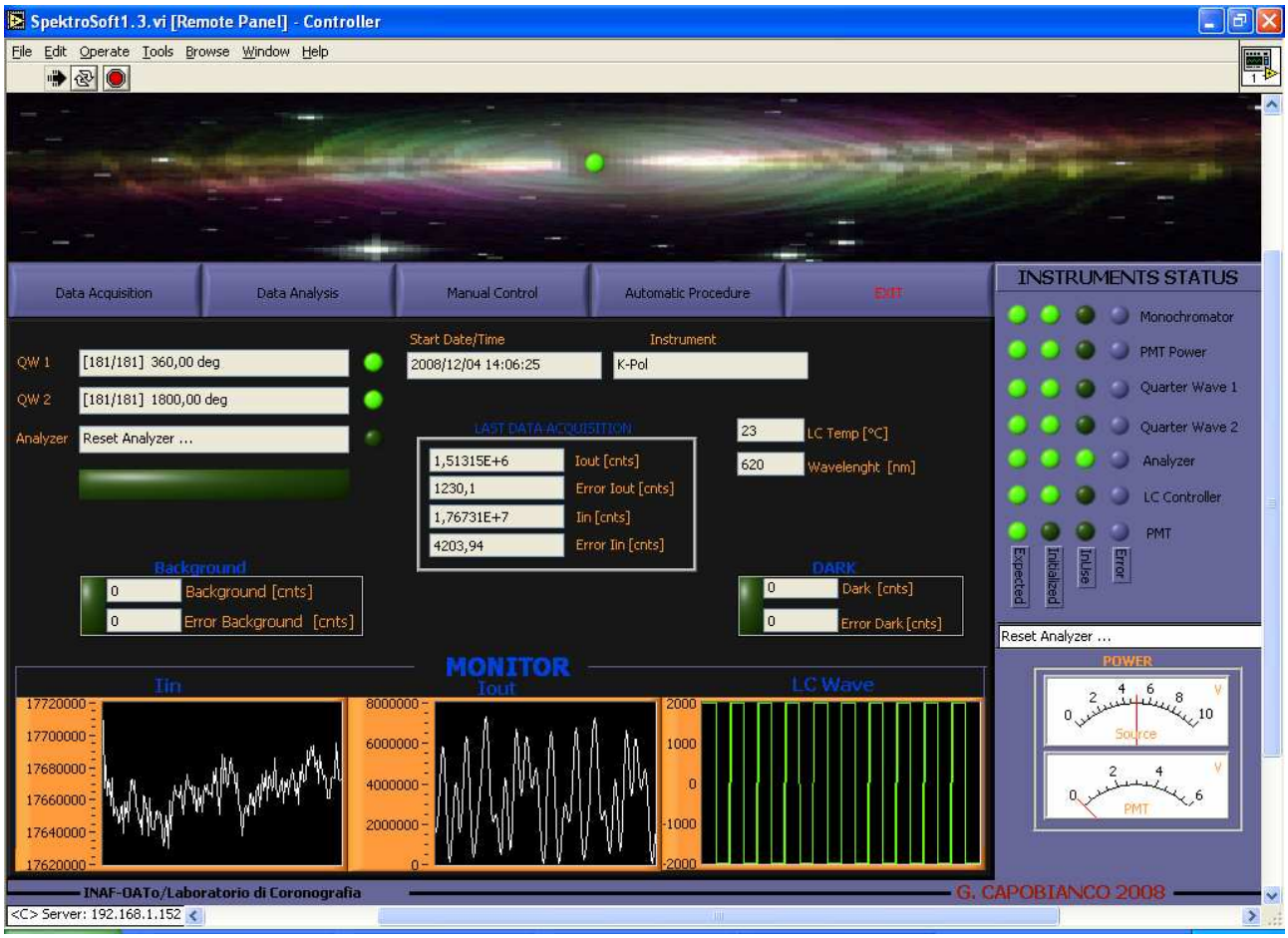


Figure 6: Screenshot of SpektroSoft.

SpektroSoft operations:

- Data acquisition: automatic data acquisition. A complete set of measurements is run using parameters set by the operator. Output is a .data file in ASCII format.
- Data Analysis: analysis of data file formatted as reported in Appendix A.
- Manual control: manual control of all instruments.
- Automatic Procedure: acquisition of a set of data like “Data Acquisition” and automatic analyze. Output is a .data file like “Data Acquisition” and a measurement report.

At the moment, the development of the data analysis tool is in progress, and this function is disabled. The modulated signal is analyzed using separate FORTRAN routines, designed for test and development of the methodology. The MSP Control code is also running on a remote PC, located outside the laboratory area, allowing a more flexible and comfortable data acquisition operations.

A detailed list of controlled instruments is given in Tab. 1.

Instrument	Description	Communication type
Monochromator	Select a wavelength	GPIB-IEEE 488
Power Supply	Photomultiplier power supply	Serial port-RS 232
Rotator 1	Motorized rotator for Quarter Wave 1	GPIB-IEEE 488
Rotator 2	Motorized rotator for Quarter Wave 2	GPIB-IEEE 488
Rotator 3	Motorized rotator for Analyzer	PCI card
LC Controller	Controller of LCs	Parallel port
Photomultiplier 1	Detector for monitoring input light	PCI card
Photomultiplier 2	Detector for monitoring output light	PCI card

Tab. 1 – List of instruments controlled by SpektroSoft.

3. Mueller Matrix Description of Dual Rotating Retarder Technique

The output signal from the MSP has a period of 180°, so after a complete rotation of the first quarter wave the information is redundantly encoded a second time. The frequency analysis of data can be performed with the Fast Fourier Transform technique, or equivalently we can use a least square method to fit the signal (see Goldstein & Chipman, 1990):

$$s_{out}(q) = I_q = \frac{a_0}{2} + \sum_{n=1}^{12} \left[a_n \cos(2n\gamma q) + b_n \sin(2n\gamma q) \right]$$

where q is the progressive number of the measures, a_n and b_n are the Fourier coefficients to be determined, which are functions of the 16 Mueller matrix elements of the sample. The systematic errors in the alignment of the setup components, i.e. the two quarter waves and the output polarizer (the transmission axis of the first polarizer defines the reference system), and the deviation of the retardances of the plates from the nominal value, are evaluated performing a series of measures with no sample (air), so we have the constrain for the resulting Mueller matrix of the sample to be unitary.

4. Data and analysis

During the preliminary MSP tests, data using known samples, such as a linear polarizer and a half wave plate, have been acquired. Then we took measurements of the LCVR of EKPol 04-579, for different applied voltages and at temperature T=23° (which is the typical laboratory temperature adopted during pre-eclipse calibrations). Finally we repeated the measures at the 4 voltages adopted during the eclipse observations, at T=30° as in the eclipse conditions. The summary of LCVR data analysis results is given in Tables 3 and 4, the LCVR retardance being given as δ/2, which can be directly compared with the resulting rotations of the assembled polarimeter, and after reducing the retardance values to the correct quadrant. The complete list of all data acquired and analysis details are also given.

Summary of all acquired data

N	File name	Date	λ[Å]	Sample	V[mV]	T[°C]	Comments
1	K-Pol-20081210-140649.data	2008/12/10 14:06:49	620	None	na	na	Misure calibrazione aria 620nm
2	K-Pol-20081211-122625.data	2008/12/11 12:26:25	620	Linear Polarizer	na	na	Misure LP 620nm
3	K-Pol-20081211-144748.data	2008/12/11 14:47:48	620	Half Wave	na	na	Misure HW 620nm
4	K-Pol-20081211-163407.data	2008/12/11 16:34:07	620	LCVR 04-579	0	23	Misure LCVR 0V 620nm
5	K-Pol-20081212-122326.data	2008/12/12 12:23:26	620	None	na	na	Misure calibrazione aria 620nm
6	K-Pol-20081212-140935.data	2008/12/12 14:09:35	620	LCVR 04-579	2000	23	Misure LCVR 04-579 2V 620nm
7	K-Pol-20081215-100938.data	2008/12/15 10:09:38	620	None	na	na	Misure calibrazione 620nm
8	K-Pol-20081215-130904.data	2008/12/15 13:09:04	620	LCVR 04-579	3000	23	Misure LCVR 04-579 3V 620nm
9	K-Pol-20081215-154142.data	2008/12/15 15:41:42	620	LCVR 04-579	4000	23	Misure LCVR 04-579 4V 620nm
10	K-Pol-20081216-103207.data	2008/12/16 10:32:07	620	LCVR 04-579	5000	23	Misure LCVR 04-579 5V 620nm
11	K-Pol-20081216-131417.data	2008/12/16 13:14:17	620	LCVR 04-579	6000	23	Misure LCVR 04-579 6V 620nm
12	K-Pol-20081216-145447.data	2008/12/16 14:54:47	620	LCVR 04-579	8000	23	Misure LCVR 04-579 8V 620nm
13	K-Pol-20081217-102954.data	2008/12/17 10:29:54	620	LCVR 04-579	10000	23	Misure LCVR 04-579 10V 620nm
14	K-Pol-20081217-142249.data	2008/12/17 14:22:49	620	LCVR 04-579	12000	23	Misure LCVR 04-579 12V 620nm
15	K-Pol-20081218-112736.data	2008/12/18 11:27:36	620	LCVR 04-579	1500	23	Misure LCVR 04-579 1.5V 620nm
16	K-Pol-20081218-132209.data	2008/12/18 13:22:09	620	LCVR 04-579	2500	23	Misure LCVR 04-579 2.5V 620nm
17	K-Pol-20081218-150234.data	2008/12/18 15:02:34	620	LCVR 04-579	15000	23	Misure LCVR 04-579 15V 620nm
18	K-Pol-20081218-164905.data	2008/12/18 16:49:05	620	None	na	na	Misure calibrazione aria 620nm
19	K-Pol-20081222-104912.data	2008/12/22 10:49:12	620	LCVR 04-579	4500	30	Misure LCVR 04-579 4.5V 620nm
20	K-Pol-20081222-123848.data	2008/12/22 12:38:48	620	LCVR 04-579	5400	30	Misure LCVR 04-579 5.4 V 620nm
21	K-Pol-20081222-144656.data	2008/12/22 14:46:56	620	LCVR 04-579	7000	23	Misure LCVR 04-579 7V 620nm
22	K-Pol-20081223-102607.data	2008/12/23 10:26:07	620	LCVR 04-579	7000	30	Misure LCVR 04-579 7V 620nm
23	K-Pol-20081223-122022.data	2008/12/23 12:20:22	620	LCVR 04-579	10000	30	Misure LCVR 04-579 10V 620nm

Tab. 2

Summary of results at the LCVR voltages applied during eclipse observations

V [mV]	MSP δ/2 Dec. 2008 T=30°	Rot Eclipse 29/03/2006 T=30°	Rot Pre-eclipse 29/03/2006 T=30°	Rot Post-eclipse 29/03/2006 T=30°	Rot Moon Mar. 2006 T unknown	Rot Lab-first set 2005 T=23°
4500.	147.25	161.	171.66	152.61	184.02	92.03
5400.	108.68	121.	130.43	114.04	139.75	
7000.	63.07	72.	79.94	68.89	87.40	32.04
10000.	26.87	33.	38.81	32.55	44.10	10.03

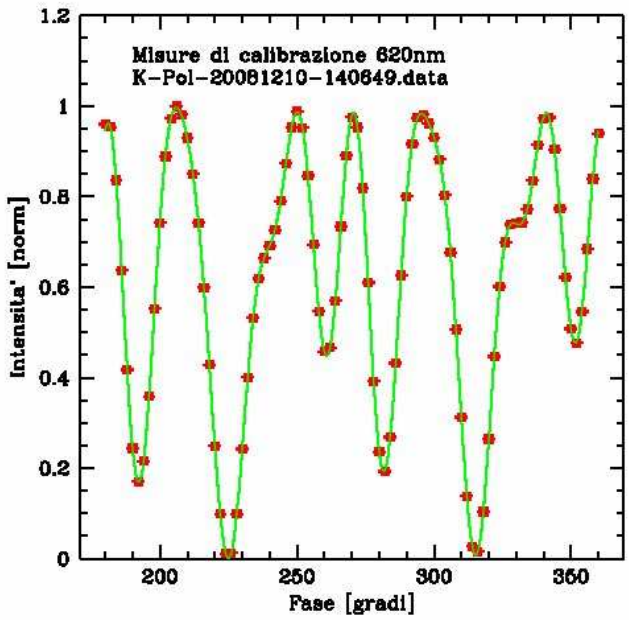
Tab. 3

Summary of MSP results at T=23°

V	δ/2
0.	328.98
1500.	328.30
2000.	323.22
2500.	296.56
3000.	254.87
4000.	185.50
5000.	132.24
6000.	90.00
7000.	71.31
8000.	51.9
10000.	29.79
12000.	17.18
15000.	4.17

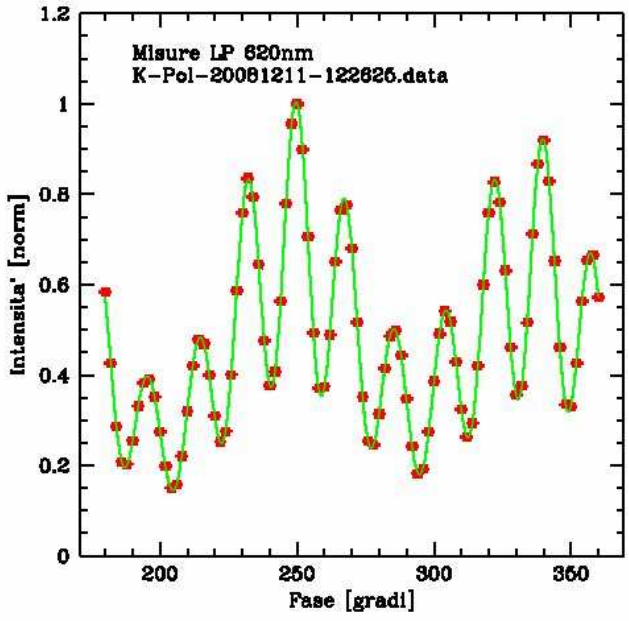
Tab. 4

Title: **Application of the Dual Rotating Retarder Polarimeter Technique to the Calibration of a Liquid Crystal Variable Retarder**



N.1 Calibration

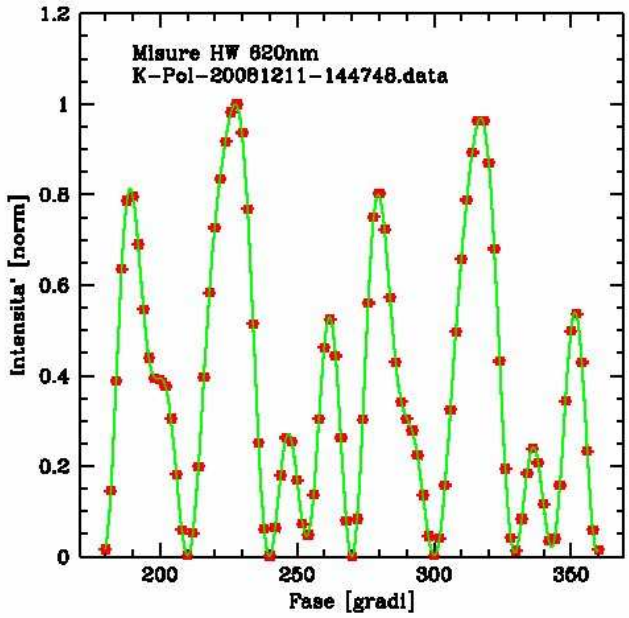
Comments: Misure aria 620nm	
File: K-Pol-20081210-140649.data	
ε_3 = 7.23 ± 0.01 [deg]	ε_4 = -2.25 ± 0.02 [deg]
ε_5 = 5.17 ± 0.04 [deg]	
δ_1 = 87.7 ± 0.2 [deg]	δ_2 = 90.2 ± 0.2 [deg]
Fit: χ^2_r = 49.33	



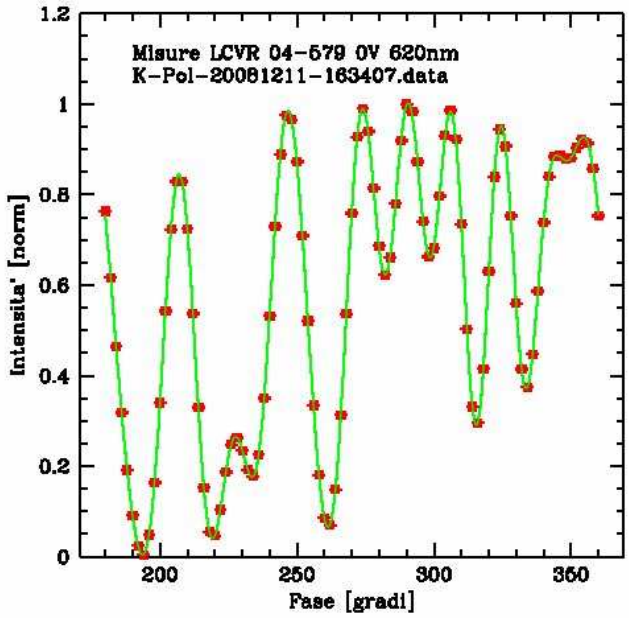
N.2 Test with a Linear Polarizer

Comments: Misure LP 620nm			
File: K-Pol-20081211-122625.data			
ε_3 = 7.23	ε_4 = -2.25	ε_5 = 5.17	
δ_1 = 87.66	δ_2 = 90.23		
Fit: χ^2_r = 7.01			
Mueller Matrix Elements			
1.	0.43267	-0.91590	-0.01401
0.42823	0.23481	-0.36429	-0.01964
-0.90415	-0.39071	0.83863	0.04518
0.00446	-0.00087	-0.02812	-0.00793
Transmission Axis Orientation			
-32.18		-33.17	

Title: **Application of the Dual Rotating Retarder Polarimeter Technique to the Calibration of a Liquid Crystal Variable Retarder**

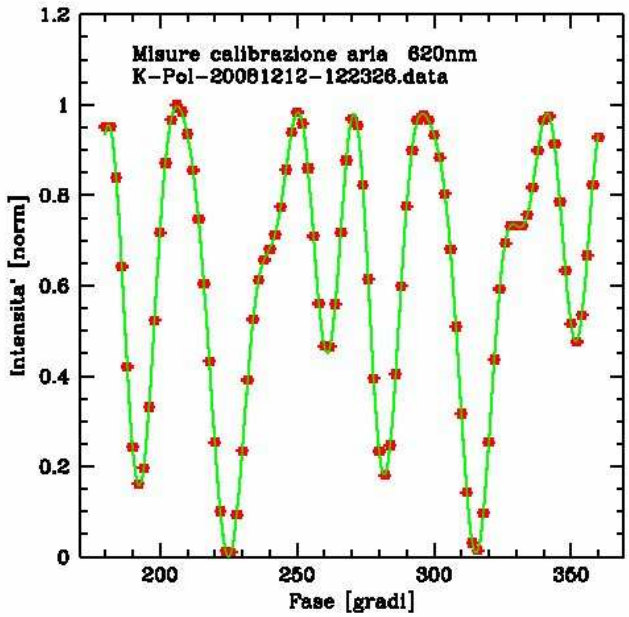


N.3 Test with HW			
Misure HW 620nm			
File: K-Pol-20081211-144748.data			
$\varepsilon_3 =$ 7.23	$\varepsilon_4 =$ -2.25	$\varepsilon_5 =$ 5.17	
$\delta_1 =$ 87.66	$\delta_2 =$ 90.23		
Fit: $\chi^2_r =$ 51.38			
Mueller Matrix Elements			
1.	0.01215	-0.00761	0.00884
0.01025	-1.01884	-0.00575	0.07179
0.04448	-0.07727	0.99733	0.02725
-0.00902	0.01212	0.02608	-0.99924
Fast Axsis			
43.95			

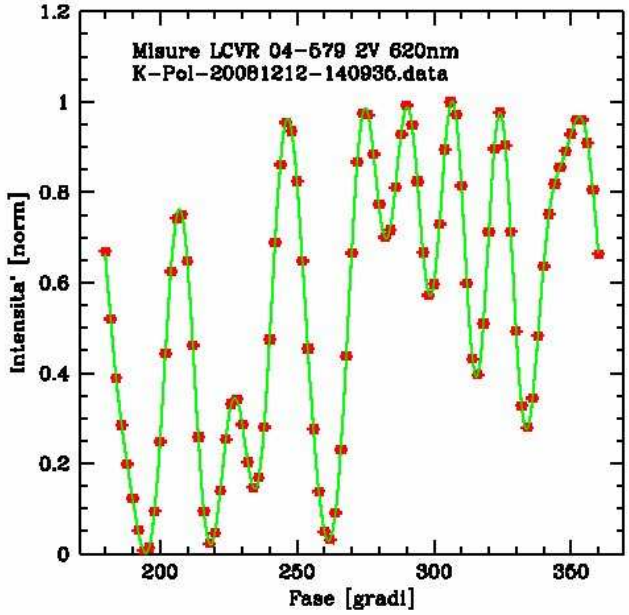


N.4 Test with LCVR at 0V			
Misure LCVR 04-579 0V 620nm			
File: K-Pol-20081212-122326.data			
$\varepsilon_3 =$ 7.23	$\varepsilon_4 =$ -2.25	$\varepsilon_5 =$ 5.17	
$\delta_1 =$ 87.66	$\delta_2 =$ 90.23		
Fit: $\chi^2_r =$ 20.75			
Mueller Matrix Elements			
1.	-0.02420	-0.00721	0.01954
-0.02728	0.57118	-0.02795	-0.88121
0.00180	-0.01373	0.99894	-0.06396
-0.00594	0.82197	0.06879	0.46885
Retardance		**Fast Axsis**	
62.04		47.23	

Title: **Application of the Dual Rotating Retarder Polarimeter Technique to the Calibration of a Liquid Crystal Variable Retarder**

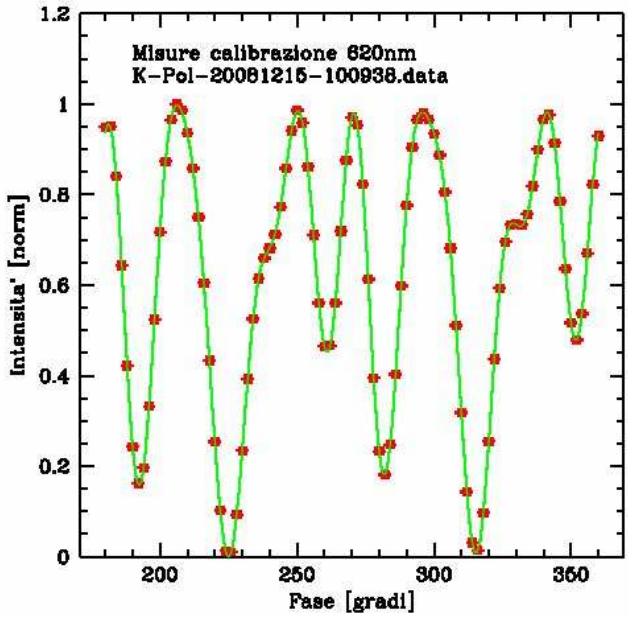


N.5 Calibration	
Comments: Misure aria 620nm	
File: K-Pol-20081212-122326.data	
ε_3 = 7.49 ± 0.01 [deg]	ε_4 = -2.90 ± 0.04 [deg]
ε_5 = 5.21 ± 0.06 [deg]	
δ_1 = 87.3 ± 0.3 [deg]	δ_2 = 90.7 ± 0.3 [deg]
Fit: χ^2_r = 24.1	

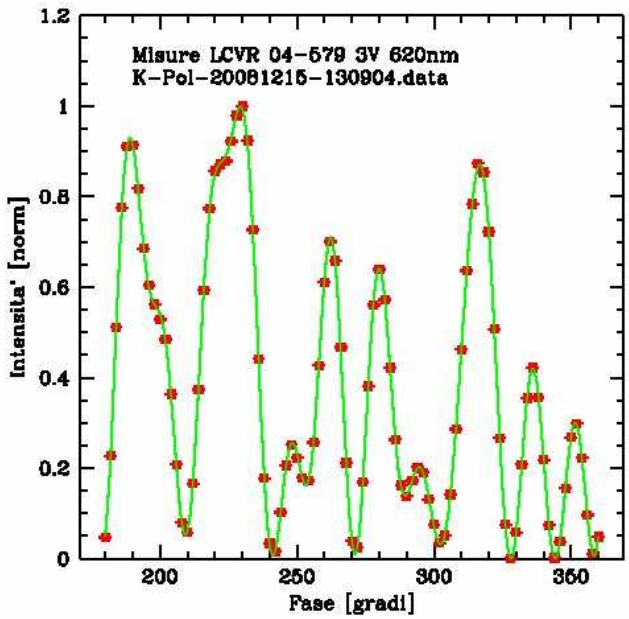


N.6 Test with LCVR at 2V			
Misure LCVR 04-579 2V 620nm			
File: K-Pol-20081212-140935.data			
ε_3 = 7.50	ε_4 = -2.90	ε_5 = 5.21	
δ_1 = 88.27	δ_2 = 90.70		
Fit: χ^2_r = 24.1			
Mueller Matrix Elements			
1.	0.01889	-0.01369	0.01432
-0.02460	0.38529	-0.03466	-0.94903
-0.00258	-0.02355	0.96318	-0.06051
0.00620	0.89999	0.06250	0.28286
Retardance		**Fast Axsis**	
73.57		46.87	

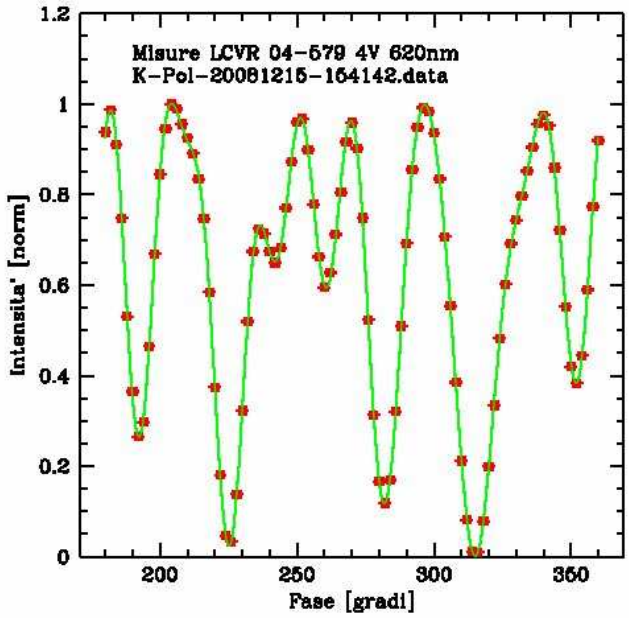
Title: **Application of the Dual Rotating Retarder Polarimeter Technique to the Calibration of a Liquid Crystal Variable Retarder**



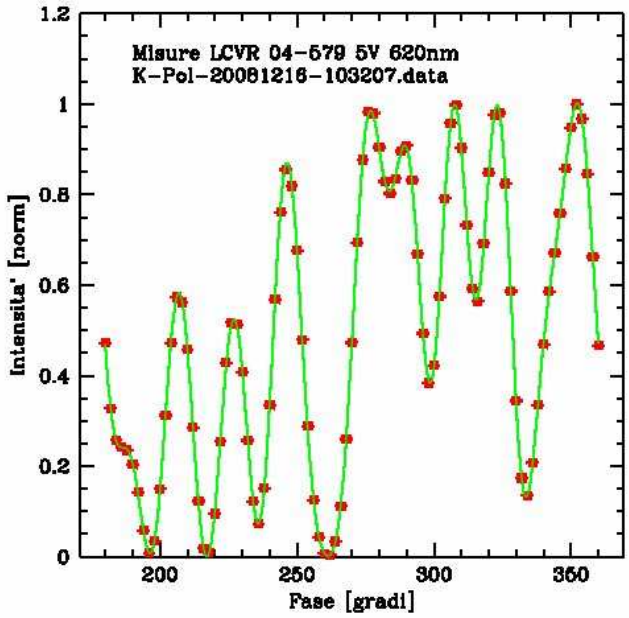
N.7 Calibration	
Comments: Misure aria 620nm	
File: K-Pol-20081215-100938.data	
$\varepsilon_3 =$ 7.54 ± 0.01 [deg]	$\varepsilon_4 =$ -2.91 ± 0.04 [deg]
$\varepsilon_5 =$ 5.24 ± 0.06 [deg]	
$\delta_1 =$ 88.3 ± 0.3 [deg]	$\delta_2 =$ 90.8 ± 0.3 [deg]
Fit: $\chi^2_r =$ 16.7	



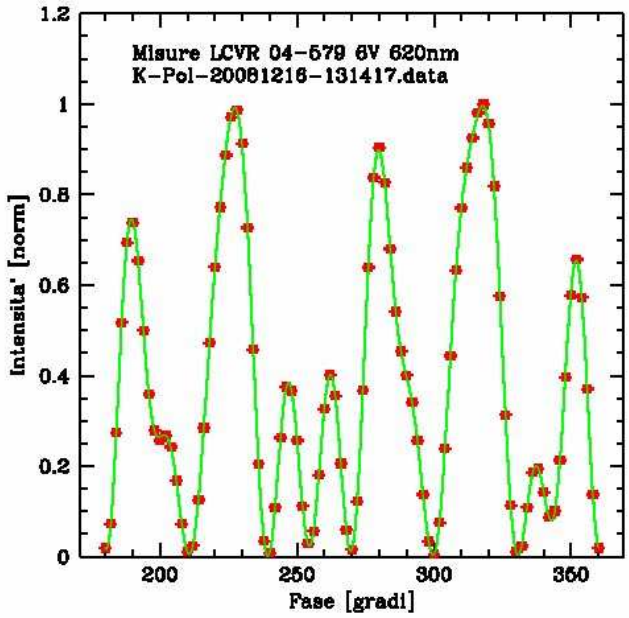
N.8 Test with LCVR at 3V			
Misure LCVR 04-579 3V 620nm			
File: K-Pol-20081215-130904.data			
$\varepsilon_3 =$ 7.54	$\varepsilon_4 =$ -2.91	$\varepsilon_5 =$ 5.24	
$\delta_1 =$ 88.27	$\delta_2 =$ 90.78		
Fit: $\chi^2_r =$ 167.34			
Mueller Matrix Elements			
1.	-0.00251	-0.00425	0.01600
-0.01306	-0.88490	-0.09999	0.47840
0.02079	-0.12519	0.96528	0.02618
-0.01286	-0.39086	0.00042	-0.86383
Retardance		**Fast Axsis**	
149.75		45.02	



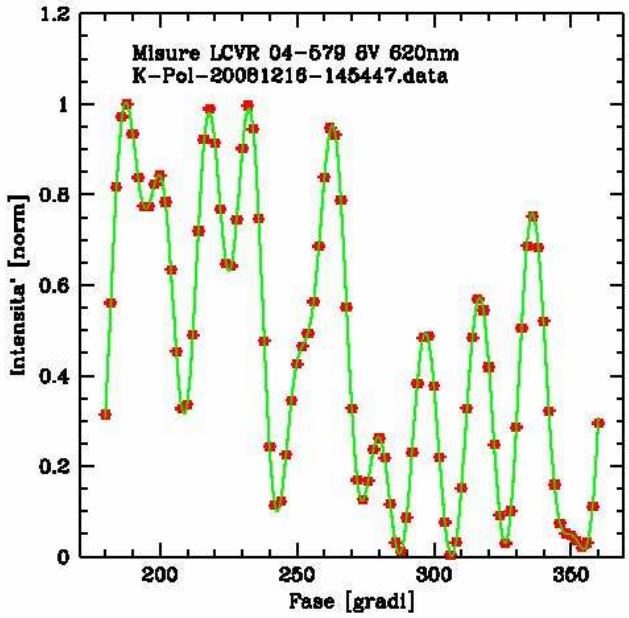
N.9 Test with LCVR at 4V			
Misure LCVR 04-579 4V 620nm			
File: K-Pol-20081215-154142.data			
ε_3 = 7.54	ε_4 = -2.91	ε_5 = 5.24	
δ_1 = 88.27	δ_2 = 90.78		
Fit: χ^2_r = 156.34			
Mueller Matrix Elements			
1.	0.04271	0.01107	0.00077
0.04221	0.87984	0.00144	0.19332
-0.00387	0.03106	1.01693	-0.00048
-0.00485	-0.24326	-0.00972	0.98046
Retardance		**Fast Axsis**	
11.34		43.58	



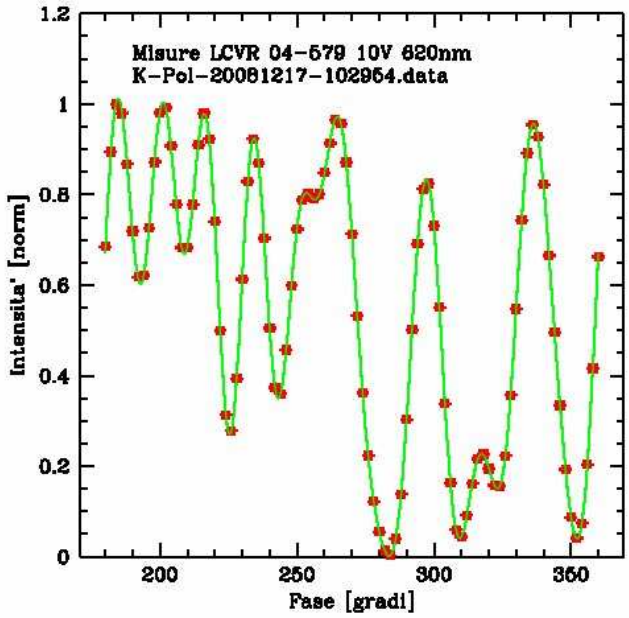
N.10 Test with LCVR at 5V			
Misure LCVR 04-579 5V 620nm			
File: K-Pol-20081215-154142.data			
ε_3 = 7.54	ε_4 = -2.91	ε_5 = 5.24	
δ_1 = 88.27	δ_2 = 90.78		
Fit: χ^2_r = 156.34			
Mueller Matrix Elements			
1.	0.04271	0.01107	0.00077
0.04221	0.87984	0.00144	0.19332
-0.00387	0.03106	1.01693	-0.00048
-0.00485	-0.24326	-0.00972	0.98046
Retardance		**Fast Axsis**	
11.34		43.58	



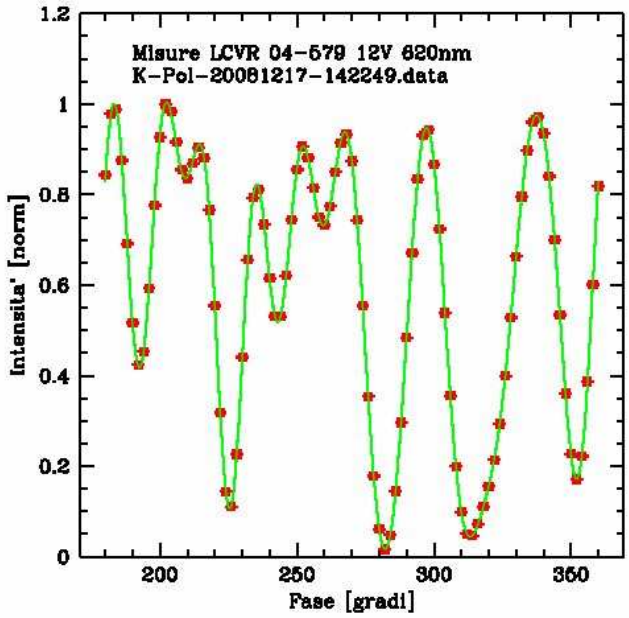
N.11 Test with LCVR at 6V			
Misure LCVR 04-579 6V 620nm			
File: K-Pol-20081216-131417.data			
ε_3 = 7.54	ε_4 = -2.91	ε_5 = 5.24	
δ_1 = 88.27	δ_2 = 90.78		
Fit: χ^2_r = 102.3			
Mueller Matrix Elements			
1.	0.01214	-0.00512	0.01123
0.01296	-0.98425	-0.13458	-0.17260
0.03461	-0.17428	0.95956	-0.02040
-0.01504	0.25964	0.01181	-1.00006
Retardance		**Fast Axsis**	
180.00		nan	



N.12 Test with LCVR at 8V			
Misure LCVR 04-579 8V 620nm			
File: K-Pol-20081216-145447.data			
ε_3 = 7.54	ε_4 = -2.91	ε_5 = 5.24	
δ_1 = 88.27	δ_2 = 90.78		
Fit: χ^2_r = 125.6			
Mueller Matrix Elements			
1.	-0.01307	0.00151	0.01009
-0.0098	-0.32038	-0.10721	0.94743
0.02105	-0.13317	0.96923	0.04840
-0.00472	-0.91019	-0.08653	-0.23906
Retardance		**Fast Axsis**	
103.83		42.44	

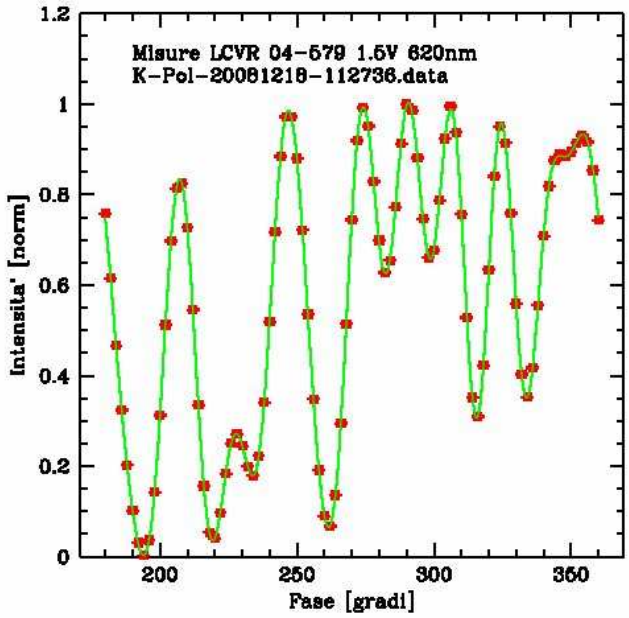


N.13 Test with LCVR at 10V			
Misure LCVR 04-579 10V 620nm			
File: K-Pol-20081217-102954.data			
ε_3 = 7.54	ε_4 = -2.91	ε_5 = 5.24	
δ_1 = 88.27	δ_2 = 90.78		
Fit: χ^2_r = 39.85			
Mueller Matrix Elements			
1.	-0.00789	0.00629	0.01024
0.00035	0.42975	-0.03233	0.85497
0.03120	-0.07859	1.01673	0.05905
0.00360	-0.89227	-0.11298	0.50619
Retardance		**Fast Axsis**	
59.59		41.23	

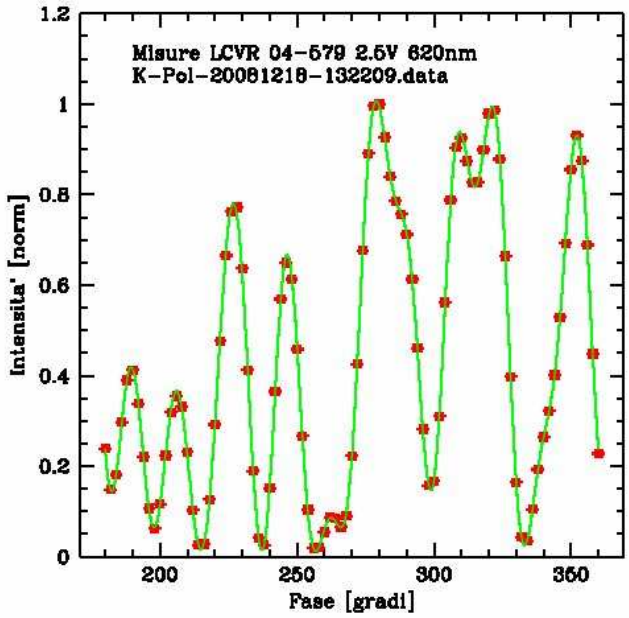


N.14 Test with LCVR at 12V			
Misure LCVR 04-579 12V 620nm			
File: K-Pol-20081217-142249.data			
ε_3 = 7.35	ε_4 = -3.07	ε_5 = 5.21	
δ_1 = 88.32	δ_2 = 90.83		
Fit: χ^2_r = 48.61			
Mueller Matrix Elements			
1.	-0.00605	-0.00217	0.00666
0.00447	0.76416	-0.00312	0.55794
0.00375	-0.00704	1.01770	0.03790
0.00661	-0.62395	-0.0645	0.82509
Retardance		**Fast Axsis**	
34.40		41.72	

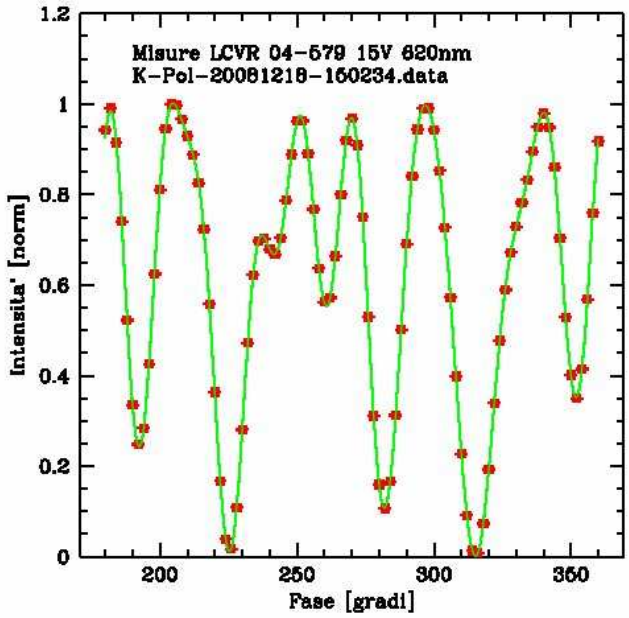
Title: **Application of the Dual Rotating Retarder Polarimeter Technique to the Calibration of a Liquid Crystal Variable Retarder**



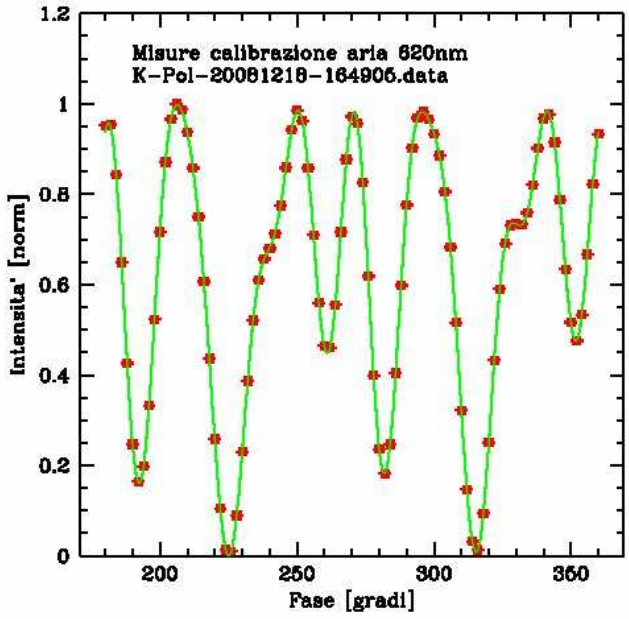
N.15 Test with LCVR at 1.5V			
Misure LCVR 04-579 1.5V 620nm			
File: K-Pol-20081218-112736.data			
ε_3 = 7.35	ε_4 = -3.07	ε_5 = 5.21	
δ_1 = 88.32	δ_2 = 90.83		
Fit: χ^2_r = 21.76			
Mueller Matrix Elements			
1.	-0.02001	-0.00741	0.00947
-0.02034	0.54068	-0.02749	-0.87892
-0.00615	-0.00292	0.97397	-0.05495
0.00649	0.82182	0.05529	0.44751
Retardance		**Fast Axsis**	
63.42		46.77	



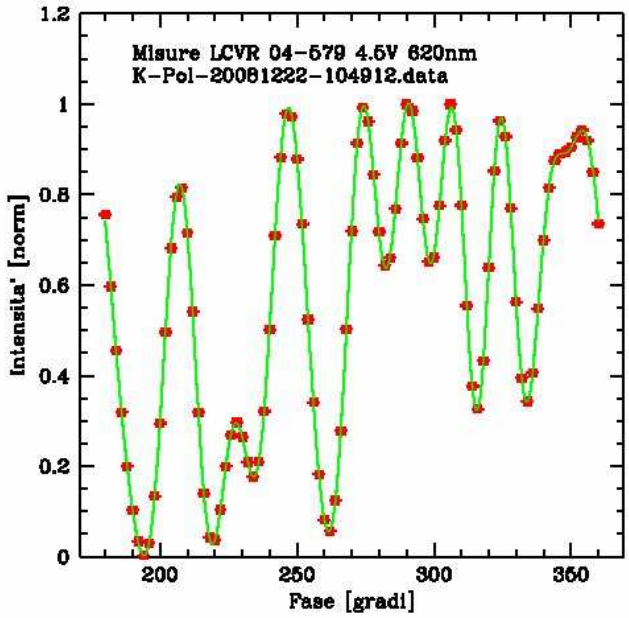
N.16 Test with LCVR at 2.5V			
Misure LCVR 04-579 2.5V 620nm			
File: K-Pol-20081218-132209.data			
ε_3 = 7.35	ε_4 = -3.07	ε_5 = 5.21	
δ_1 = 88.32	δ_2 = 90.83		
Fit: χ^2_r = 187.36			
Mueller Matrix Elements			
1.	-0.03037	-0.00526	0.01380
-0.04059	-0.46387	-0.10209	-0.78337
0.02185	-0.12756	0.91708	-0.05114
0.00623	0.80816	0.05584	-0.60012
Retardance		**Fast Axsis**	
126.88		47.00	



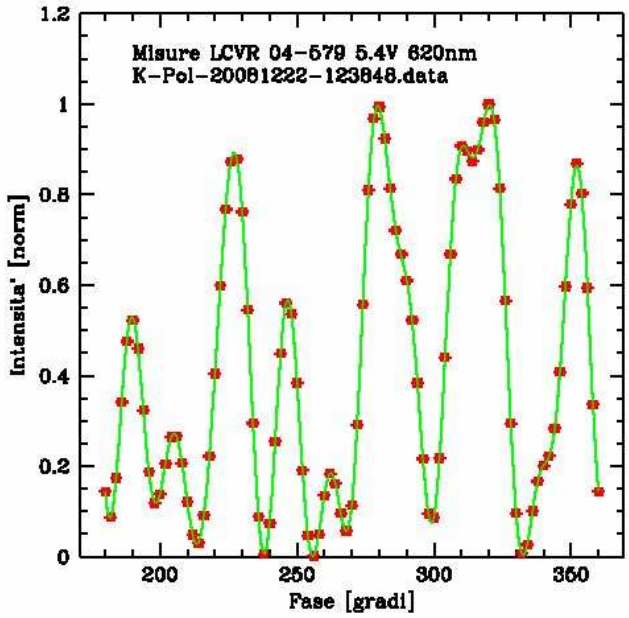
N.17 Test with LCVR at 15V			
Misure LCVR 04-579 15V 620nm			
File: K-Pol-20081218-150234.data			
ε_3 = 7.35	ε_4 = -3.07	ε_5 = 5.21	
δ_1 = 88.32	δ_2 = 90.83		
Fit: χ^2_r= 30.02			
Mueller Matrix Elements			
1.	-0.01637	0.00386	0.00527
-0.00140	0.96148	0.01126	0.14781
0.00598	0.01223	1.02101	0.01913
0.00968	-0.23761	-0.03364	0.98910
Retardance		**Fast Axsis**	
8.47		38.40	



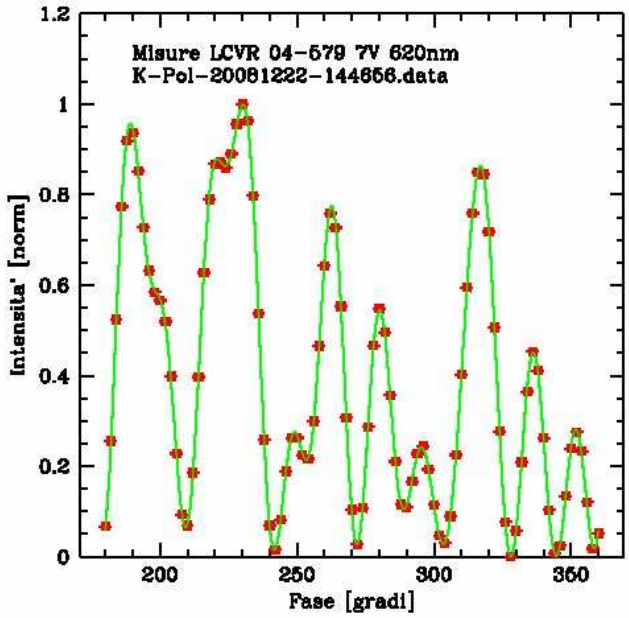
N.18 Calibration	
Comments: Misure aria 620nm	
File: K-Pol-20081218-164905.data	
ε_3 = 7.35 ± 0.01 [deg]	ε_4 = -3.07 ± 0.05 [deg]
ε_5 = 5.21 ± 0.07 [deg]	
δ_1 = 88.3 ± 0.3 [deg]	δ_2 = 90.8 ± 0.3 [deg]
Fit: χ^2_r= 18.30	



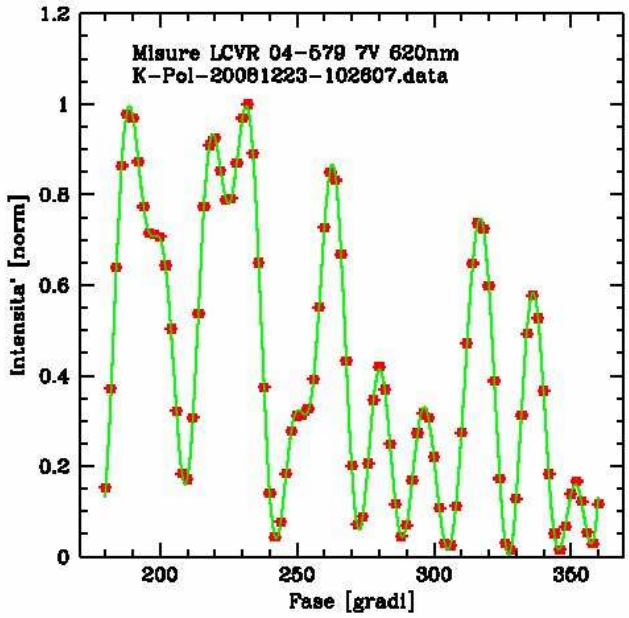
N.19 Test with LCVR at 4.5V			
Misure LCVR 04-579 4.5V 620nm			
File: K-Pol-20081222-104912.data			
$\varepsilon_3 =$ 7.35	$\varepsilon_4 =$ -3.07	$\varepsilon_5 =$ 5.21	
$\delta_1 =$ 88.32	$\delta_2 =$ 90.83		
Fit: $\chi^2_r =$ 61.90			
Mueller Matrix Elements			
1.	-0.02893	-0.01155	0.01054
-0.03349	0.53385	-0.04784	-0.88825
0.00212	-0.01401	0.98392	-0.08261
-0.00190	0.84267	0.06187	0.41469
Retardance		**Fast Axsis**	
65.50		46.95	



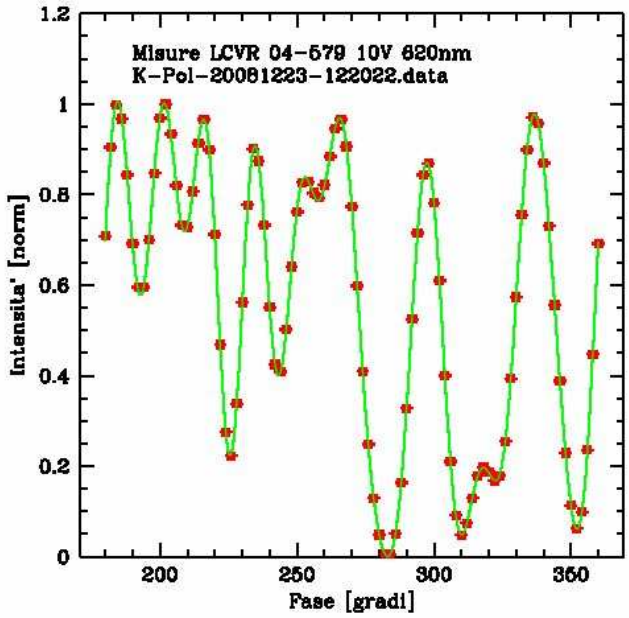
N.20 Test with LCVR at 5.4V			
Misure LCVR 04-579 5.4V 620nm			
File: K-Pol-20081222-123848.data			
$\varepsilon_3 =$ 7.35	$\varepsilon_4 =$ -3.07	$\varepsilon_5 =$ 5.21	
$\delta_1 =$ 88.32	$\delta_2 =$ 90.83		
Fit: $\chi^2_r =$ 51.31			
Mueller Matrix Elements			
1.	-0.00774	-0.00208	0.01611
-0.00700	-0.71298	-0.14449	-0.61188
0.01432	-0.16124	0.93754	-0.06396
-0.01898	0.68373	0.04801	-0.79469
Retardance		**Fast Axsis**	
142.63		47.27	



N.21 Test with LCVR at 7V			
Misure LCVR 04-579 7V 620nm			
File: K-Pol-20081222-144656.data			
ε_3 = 7.35	ε_4 = -3.07	ε_5 = 5.21	
δ_1 = 88.32	δ_2 = 90.83		
Fit: χ^2_r = 305.46			
Mueller Matrix Elements			
1.	0.01648	-0.00109	0.01093
-0.00759	-0.83843	-0.18677	0.57729
0.03483	-0.22248	-0.09276	-0.05156
-0.01082	-0.49653	-0.09276	-0.81285
Retardance		**Fast Axsis**	
144.38		40.42	



N.22 Test with LCVR at 7V			
Misure LCVR 04-579 7V 620nm			
File: K-Pol-20081223-102607.data			
ε_3 = 7.35	ε_4 = -3.07	ε_5 = 5.21	
δ_1 = 88.32	δ_2 = 90.83		
Fit: χ^2_r = 377.92			
Mueller Matrix Elements			
1.	0.00446	0.02166	0.01472
0.00197	-0.66351	-0.18322	0.78594
0.03648	-0.22520	0.96428	0.07801
-0.00355	-0.71859	-0.10387	-0.58987
Retardance		**Fast Axsis**	
126.15		41.30	



N.23 Test with LCVR at 10V			
Misure LCVR 04-579 10V 620nm			
File: K-Pol-20081223-122022.data			
$\varepsilon_3 =$ 7.35	$\varepsilon_4 =$ -3.07	$\varepsilon_5 =$ 5.21	
$\delta_1 =$ 88.32	$\delta_2 =$ 90.83		
Fit: $\chi^2_r =$ 39.19			
Mueller Matrix Elements			
1.	-0.00871	0.01678	0.00931
-0.00722	0.52125	-0.06347	0.79093
0.01695	-0.06080	0.99366	0.07715
0.01015	-0.84730	-0.11921	0.59146
Retardance		**Fast Axsis**	
53.74		40.75	

APPENDIX A -DEFINITION OF MUELLER SPECTROPOLARIMETER DATA FILES

Data file is structured as follow:

- Header;
- Tabulation;
- Data
- Date of file creation.

Fist 15 rows of data file are designed for header. A list of keywords and values is reported in table1. Every row is delimited by "EOL" character.

Keyword	Type	Description	Units	Example
Date	S*19	Date/Time when program is runned		2008/11/03 12:38:05
Wave	I*3	Wavelength	nm	550
Bandpass	I*2	Band pass	nm	6
Source	F*1.1	Voltage applied to light source	V	6.3
Sample	S*16	Sample type		Linear Polarizer
LCVRV	I*5	Voltage applied to LCVR	mV	10000
LCVRT	I*2.2	Temperature of LCVR	°C	23
ExpTime	I*5	exposition time of photomultiplier	ms	10000
Dark	F*4.2	Dark counts	counts	700.20
DarkErr	F*4.2	Error on evaluation of dark	counts	100.38
Bkgr	F*4.2	Background	counts	300.56
BkgrErr	F*4.2	Error on evaluation of background	counts	87.98
Instrument	S*10	Instrument		KPol
Operator	S*30	Name of Operator		G. Capobianco
Comments	S	Comments		Calibration of set-up

Table A.1 – *List of keywords.*

Structure is: Keyword[white space]= [white space]Value[EOL].

Row number 16 is designed for tabulation. Structure is:

```
"#"[tab]"Date/Time"[tab]"Analyzer[deg]"[tab]"QW1[deg]"[tab]"QW2[deg]"[tab]
"Iout[cnts]"[tab]"ErrIout[cnts]"[tab]"Iin[cnts]"[tab]"ErrIin[cnts]"[EOL]
```

This structure is the same for data.

File name is automatically generated as follow:

InstrumentName "-" date-time ".data"

For example: KPol-20081103-142734.data

Path where file is saved is: ../Data.

Date = 2008/12/11 12:26:25
Wave = 620
Bandpass = 2
Source = 5,0
Sample = Linear Polarizer
LCVRV = NaN
LCVRT = NaN
ExpTime = 10000
Dark = 0,00
DarkErr = 0,00
Bkgr = 0,00
BkgrErr = 0,00
Instrument = K-Pol
Operator = Lab Team
Comments = Misure LP 620nm

#	Date/Time	Analyzer[deg]	QW1[deg]		QW2[deg]	Iout[cnts]	ErrIout[cnts]	Iin[cnts]
	ErrIin[cnts]							
1	2008/12/11 12:46:58	0,00	0,00	0,00	243540,00	493,50	1408150,00	1186,65
2	2008/12/11 12:47:31	0,00	2,00	10,00	181790,00	426,37	1410670,00	1187,72
3	2008/12/11 12:48:03	0,00	4,00	20,00	120790,00	347,55	1407860,00	1186,53
4	2008/12/11 12:48:36	0,00	6,00	30,00	85670,00	292,69	1405920,00	1185,71
5	2008/12/11 12:49:08	0,00	8,00	40,00	83890,00	289,64	1408220,00	1186,68
6	2008/12/11 12:49:41	0,00	10,00	50,00	107340,00	327,63	1403520,00	1184,70
7	2008/12/11 12:50:13	0,00	12,00	60,00	139520,00	373,52	1403760,00	1184,80
8	2008/12/11 12:50:46	0,00	14,00	70,00	160780,00	400,97	1402680,00	1184,35
9	2008/12/11 12:51:18	0,00	16,00	80,00	162890,00	403,60	1401820,00	1183,98
10	2008/12/11 12:51:51	0,00	18,00	90,00	142710,00	377,77	1402620,00	1184,32
11	2008/12/11 12:52:23	0,00	20,00	100,00	111180,00	333,44	1395440,00	1181,29
12	2008/12/11 12:52:56	0,00	22,00	110,00	78760,00	280,64	1396810,00	1181,87
13	2008/12/11 12:53:28	0,00	24,00	120,00	61520,00	248,03	1400060,00	1183,24
14	2008/12/11 12:54:01	0,00	26,00	130,00	67530,00	259,87	1399130,00	1182,85
15	2008/12/11 12:54:33	0,00	28,00	140,00	94610,00	307,59	1396980,00	1181,94
16	2008/12/11 12:55:06	0,00	30,00	150,00	136770,00	369,82	1394230,00	1180,78
17	2008/12/11 12:55:38	0,00	32,00	160,00	178230,00	422,17	1396190,00	1181,60
18	2008/12/11 12:56:11	0,00	34,00	170,00	199450,00	446,60	1395590,00	1181,35
19	2008/12/11 12:56:43	0,00	36,00	180,00	194590,00	441,12	1393320,00	1180,39
20	2008/12/11 12:57:16	0,00	38,00	190,00	164970,00	406,16	1395160,00	1181,17

21	2008/12/11 12:57:49	0,00	40,00	200,00	126830,00	356,13	1393060,00	1180,28
22	2008/12/11 12:58:21	0,00	42,00	210,00	105140,00	324,25	1392540,00	1180,06
23	2008/12/11 12:58:54	0,00	44,00	220,00	115410,00	339,72	1389950,00	1178,96
24	2008/12/11 12:59:26	0,00	46,00	230,00	165330,00	406,61	1390540,00	1179,21
25	2008/12/11 12:59:59	0,00	48,00	240,00	238670,00	488,54	1384370,00	1176,59
26	2008/12/11 13:00:31	0,00	50,00	250,00	311740,00	558,34	1387520,00	1177,93
27	2008/12/11 13:01:04	0,00	52,00	260,00	347460,00	589,46	1381670,00	1175,44
28	2008/12/11 13:01:36	0,00	54,00	270,00	335200,00	578,96	1386080,00	1177,32
29	2008/12/11 13:02:09	0,00	56,00	280,00	277900,00	527,16	1386160,00	1177,35
30	2008/12/11 13:02:41	0,00	58,00	290,00	208320,00	456,42	1383450,00	1176,20
31	2008/12/11 13:03:14	0,00	60,00	300,00	161710,00	402,13	1385520,00	1177,08
32	2008/12/11 13:03:46	0,00	62,00	310,00	168190,00	410,11	1384360,00	1176,59
33	2008/12/11 13:04:19	0,00	64,00	320,00	228350,00	477,86	1386290,00	1177,41
34	2008/12/11 13:04:51	0,00	66,00	330,00	319730,00	565,45	1384480,00	1176,64
35	2008/12/11 13:05:24	0,00	68,00	340,00	394140,00	627,81	1380330,00	1174,87
36	2008/12/11 13:05:56	0,00	70,00	350,00	420550,00	648,50	1381710,00	1175,46
37	2008/12/11 13:06:29	0,00	72,00	360,00	382030,00	618,09	1381370,00	1175,32
38	2008/12/11 13:07:01	0,00	74,00	370,00	298780,00	546,61	1382440,00	1175,77
39	2008/12/11 13:07:34	0,00	76,00	380,00	209530,00	457,74	1384570,00	1176,68
40	2008/12/11 13:08:07	0,00	78,00	390,00	154060,00	392,50	1380220,00	1174,83
41	2008/12/11 13:08:39	0,00	80,00	400,00	156550,00	395,66	1381170,00	1175,23
42	2008/12/11 13:09:12	0,00	82,00	410,00	208190,00	456,28	1382190,00	1175,67
43	2008/12/11 13:09:44	0,00	84,00	420,00	277110,00	526,41	1380130,00	1174,79
44	2008/12/11 13:10:17	0,00	86,00	430,00	324890,00	569,99	1382460,00	1175,78
45	2008/12/11 13:10:49	0,00	88,00	440,00	326290,00	571,22	1379580,00	1174,56
46	2008/12/11 13:11:22	0,00	90,00	450,00	281770,00	530,82	1378990,00	1174,30
47	2008/12/11 13:11:54	0,00	92,00	460,00	210570,00	458,88	1374890,00	1172,56
48	2008/12/11 13:12:27	0,00	94,00	470,00	143610,00	378,96	1374780,00	1172,51
49	2008/12/11 13:12:59	0,00	96,00	480,00	103460,00	321,65	1375370,00	1172,76
50	2008/12/11 13:13:32	0,00	98,00	490,00	104990,00	324,02	1376750,00	1173,35
51	2008/12/11 13:14:04	0,00	100,00	500,00	135490,00	368,09	1377040,00	1173,47
52	2008/12/11 13:14:37	0,00	102,00	510,00	176260,00	419,83	1375090,00	1172,64
53	2008/12/11 13:15:09	0,00	104,00	520,00	206310,00	454,21	1373300,00	1171,88
54	2008/12/11 13:15:42	0,00	106,00	530,00	207930,00	455,99	1375820,00	1172,95
55	2008/12/11 13:16:14	0,00	108,00	540,00	184670,00	429,73	1376530,00	1173,26
56	2008/12/11 13:16:47	0,00	110,00	550,00	141040,00	375,55	1376010,00	1173,03
57	2008/12/11 13:17:19	0,00	112,00	560,00	101370,00	318,39	1376820,00	1173,38
58	2008/12/11 13:17:52	0,00	114,00	570,00	76480,00	276,55	1374980,00	1172,60
59	2008/12/11 13:18:25	0,00	116,00	580,00	80710,00	284,10	1376660,00	1173,31
60	2008/12/11 13:18:57	0,00	118,00	590,00	111520,00	333,95	1372170,00	1171,40
61	2008/12/11 13:19:30	0,00	120,00	600,00	158170,00	397,71	1375300,00	1172,73
62	2008/12/11 13:20:02	0,00	122,00	610,00	202380,00	449,87	1372650,00	1171,60
63	2008/12/11 13:20:35	0,00	124,00	620,00	226990,00	476,43	1373850,00	1172,11
64	2008/12/11 13:21:07	0,00	126,00	630,00	217760,00	466,65	1375340,00	1172,75
65	2008/12/11 13:21:40	0,00	128,00	640,00	184750,00	429,83	1371950,00	1171,30
66	2008/12/11 13:22:12	0,00	130,00	650,00	141450,00	376,10	1371200,00	1170,98

67	2008/12/11 13:22:45	0,00	132,00	660,00	112400,00	335,26	1366820,00	1169,11
68	2008/12/11 13:23:17	0,00	134,00	670,00	120200,00	346,70	1369620,00	1170,31
69	2008/12/11 13:23:50	0,00	136,00	680,00	171650,00	414,31	1367060,00	1169,21
70	2008/12/11 13:24:22	0,00	138,00	690,00	246900,00	496,89	1370490,00	1170,68
71	2008/12/11 13:24:55	0,00	140,00	700,00	316510,00	562,59	1371850,00	1171,26
72	2008/12/11 13:25:27	0,00	142,00	710,00	351570,00	592,93	1369520,00	1170,26
73	2008/12/11 13:26:00	0,00	144,00	720,00	329870,00	574,34	1371020,00	1170,91
74	2008/12/11 13:26:32	0,00	146,00	730,00	269450,00	519,09	1370830,00	1170,82
75	2008/12/11 13:27:05	0,00	148,00	740,00	196350,00	443,11	1371150,00	1170,96
76	2008/12/11 13:27:37	0,00	150,00	750,00	151240,00	388,90	1369020,00	1170,05
77	2008/12/11 13:28:10	0,00	152,00	760,00	160100,00	400,12	1371100,00	1170,94
78	2008/12/11 13:28:43	0,00	154,00	770,00	221310,00	470,44	1370200,00	1170,56
79	2008/12/11 13:29:15	0,00	156,00	780,00	306300,00	553,44	1372730,00	1171,64
80	2008/12/11 13:29:48	0,00	158,00	790,00	372540,00	610,36	1370440,00	1170,66
81	2008/12/11 13:30:20	0,00	160,00	800,00	389410,00	624,03	1367840,00	1169,55
82	2008/12/11 13:30:53	0,00	162,00	810,00	351100,00	592,54	1369580,00	1170,29
83	2008/12/11 13:31:25	0,00	164,00	820,00	269340,00	518,98	1367490,00	1169,40
84	2008/12/11 13:31:58	0,00	166,00	830,00	187950,00	433,53	1369160,00	1170,11
85	2008/12/11 13:32:30	0,00	168,00	840,00	140430,00	374,74	1367440,00	1169,38
86	2008/12/11 13:33:03	0,00	170,00	850,00	143770,00	379,17	1367530,00	1169,41
87	2008/12/11 13:33:35	0,00	172,00	860,00	188580,00	434,26	1372940,00	1171,73
88	2008/12/11 13:34:08	0,00	174,00	870,00	244310,00	494,28	1369570,00	1170,29
89	2008/12/11 13:34:40	0,00	176,00	880,00	281940,00	530,98	1367810,00	1169,53
90	2008/12/11 13:35:13	0,00	178,00	890,00	282460,00	531,47	1364250,00	1168,01
91	2008/12/11 13:35:45	0,00	180,00	900,00	240740,00	490,65	1363990,00	1167,90
92	2008/12/11 13:36:18	0,00	182,00	910,00	176720,00	420,38	1367600,00	1169,44
93	2008/12/11 13:36:51	0,00	184,00	920,00	118680,00	344,50	1368840,00	1169,97
94	2008/12/11 13:37:23	0,00	186,00	930,00	86170,00	293,55	1365190,00	1168,41
95	2008/12/11 13:37:56	0,00	188,00	940,00	83710,00	289,33	1364950,00	1168,31
96	2008/12/11 13:38:28	0,00	190,00	950,00	105300,00	324,50	1363100,00	1167,52
97	2008/12/11 13:39:01	0,00	192,00	960,00	136880,00	369,97	1365110,00	1168,38
98	2008/12/11 13:39:33	0,00	194,00	970,00	158570,00	398,21	1366700,00	1169,06
99	2008/12/11 13:40:06	0,00	196,00	980,00	161950,00	402,43	1363070,00	1167,51
100	2008/12/11 13:40:38	0,00	198,00	990,00	145330,00	381,22	1363350,00	1167,63
101	2008/12/11 13:41:11	0,00	200,00	1000,00	113180,00	336,42	1364850,00	1168,27
102	2008/12/11 13:41:43	0,00	202,00	1010,00	81800,00	286,01	1361210,00	1166,71
103	2008/12/11 13:42:16	0,00	204,00	1020,00	62150,00	249,30	1363740,00	1167,79
104	2008/12/11 13:42:48	0,00	206,00	1030,00	65470,00	255,87	1366000,00	1168,76
105	2008/12/11 13:43:21	0,00	208,00	1040,00	91160,00	301,93	1364290,00	1168,03
106	2008/12/11 13:43:53	0,00	210,00	1050,00	132020,00	363,35	1362590,00	1167,30
107	2008/12/11 13:44:26	0,00	212,00	1060,00	173550,00	416,59	1361410,00	1166,79
108	2008/12/11 13:44:58	0,00	214,00	1070,00	197080,00	443,94	1361590,00	1166,87
109	2008/12/11 13:45:31	0,00	216,00	1080,00	193440,00	439,82	1362830,00	1167,40
110	2008/12/11 13:46:03	0,00	218,00	1090,00	164790,00	405,94	1357920,00	1165,30
111	2008/12/11 13:46:36	0,00	220,00	1100,00	127010,00	356,38	1356580,00	1164,72
112	2008/12/11 13:47:08	0,00	222,00	1110,00	104130,00	322,69	1358990,00	1165,76

113	2008/12/11 13:47:41	0,00	224,00	1120,00	113460,00	336,84	1360160,00	1166,26
114	2008/12/11 13:48:14	0,00	226,00	1130,00	165040,00	406,25	1359410,00	1165,94
115	2008/12/11 13:48:46	0,00	228,00	1140,00	240340,00	490,24	1354990,00	1164,04
116	2008/12/11 13:49:19	0,00	230,00	1150,00	311050,00	557,72	1353970,00	1163,60
117	2008/12/11 13:49:51	0,00	232,00	1160,00	343610,00	586,18	1357880,00	1165,28
118	2008/12/11 13:50:24	0,00	234,00	1170,00	326800,00	571,66	1359590,00	1166,01
119	2008/12/11 13:50:56	0,00	236,00	1180,00	265160,00	514,94	1359320,00	1165,90
120	2008/12/11 13:51:29	0,00	238,00	1190,00	195970,00	442,68	1357850,00	1165,27
121	2008/12/11 13:52:01	0,00	240,00	1200,00	154920,00	393,60	1357660,00	1165,19
122	2008/12/11 13:52:34	0,00	242,00	1210,00	167530,00	409,30	1357530,00	1165,13
123	2008/12/11 13:53:06	0,00	244,00	1220,00	231760,00	481,41	1355350,00	1164,20
124	2008/12/11 13:53:39	0,00	246,00	1230,00	321070,00	566,63	1359290,00	1165,89
125	2008/12/11 13:54:11	0,00	248,00	1240,00	393010,00	626,91	1358430,00	1165,52
126	2008/12/11 13:54:44	0,00	250,00	1250,00	410500,00	640,70	1356350,00	1164,62
127	2008/12/11 13:55:16	0,00	252,00	1260,00	369080,00	607,52	1357090,00	1164,94
128	2008/12/11 13:55:49	0,00	254,00	1270,00	289710,00	538,25	1355300,00	1164,17
129	2008/12/11 13:56:21	0,00	256,00	1280,00	202620,00	450,13	1356840,00	1164,83
130	2008/12/11 13:56:54	0,00	258,00	1290,00	152440,00	390,44	1354600,00	1163,87
131	2008/12/11 13:57:27	0,00	260,00	1300,00	153700,00	392,05	1353150,00	1163,25
132	2008/12/11 13:57:59	0,00	262,00	1310,00	201160,00	448,51	1358280,00	1165,45
133	2008/12/11 13:58:32	0,00	264,00	1320,00	267350,00	517,06	1354050,00	1163,64
134	2008/12/11 13:59:04	0,00	266,00	1330,00	313800,00	560,18	1355430,00	1164,23
135	2008/12/11 13:59:37	0,00	268,00	1340,00	318480,00	564,34	1355450,00	1164,24
136	2008/12/11 14:00:09	0,00	270,00	1350,00	279170,00	528,37	1357040,00	1164,92
137	2008/12/11 14:00:42	0,00	272,00	1360,00	212120,00	460,56	1355170,00	1164,12
138	2008/12/11 14:01:14	0,00	274,00	1370,00	144420,00	380,03	1355060,00	1164,07
139	2008/12/11 14:01:47	0,00	276,00	1380,00	104090,00	322,63	1354200,00	1163,70
140	2008/12/11 14:02:19	0,00	278,00	1390,00	100970,00	317,76	1359850,00	1166,13
141	2008/12/11 14:02:52	0,00	280,00	1400,00	129220,00	359,47	1357740,00	1165,22
142	2008/12/11 14:03:24	0,00	282,00	1410,00	169410,00	411,59	1352600,00	1163,01
143	2008/12/11 14:03:57	0,00	284,00	1420,00	199110,00	446,22	1353000,00	1163,19
144	2008/12/11 14:04:29	0,00	286,00	1430,00	204760,00	452,50	1353660,00	1163,47
145	2008/12/11 14:05:02	0,00	288,00	1440,00	182580,00	427,29	1357950,00	1165,31
146	2008/12/11 14:05:34	0,00	290,00	1450,00	142570,00	377,58	1355510,00	1164,26
147	2008/12/11 14:06:07	0,00	292,00	1460,00	99480,00	315,40	1357310,00	1165,04
148	2008/12/11 14:06:39	0,00	294,00	1470,00	75050,00	273,95	1356580,00	1164,72
149	2008/12/11 14:07:12	0,00	296,00	1480,00	79020,00	281,10	1354690,00	1163,91
150	2008/12/11 14:07:45	0,00	298,00	1490,00	112260,00	335,05	1353570,00	1163,43
151	2008/12/11 14:08:17	0,00	300,00	1500,00	158850,00	398,56	1356910,00	1164,86
152	2008/12/11 14:08:50	0,00	302,00	1510,00	202220,00	449,69	1358220,00	1165,43
153	2008/12/11 14:09:22	0,00	304,00	1520,00	221780,00	470,94	1353510,00	1163,40
154	2008/12/11 14:09:55	0,00	306,00	1530,00	213020,00	461,54	1358170,00	1165,41
155	2008/12/11 14:10:27	0,00	308,00	1540,00	175340,00	418,74	1351230,00	1162,42
156	2008/12/11 14:11:00	0,00	310,00	1550,00	133380,00	365,21	1354840,00	1163,98
157	2008/12/11 14:11:32	0,00	312,00	1560,00	108160,00	328,88	1356420,00	1164,65
158	2008/12/11 14:12:05	0,00	314,00	1570,00	120590,00	347,26	1357680,00	1165,20

159	2008/12/11 14:12:37	0,00	316,00	1580,00	172760,00	415,64	1355870,00	1164,42
160	2008/12/11 14:13:10	0,00	318,00	1590,00	245670,00	495,65	1352000,00	1162,76
161	2008/12/11 14:13:42	0,00	320,00	1600,00	310540,00	557,26	1352150,00	1162,82
162	2008/12/11 14:14:15	0,00	322,00	1610,00	338680,00	581,96	1351970,00	1162,74
163	2008/12/11 14:14:47	0,00	324,00	1620,00	320640,00	566,25	1353150,00	1163,25
164	2008/12/11 14:15:20	0,00	326,00	1630,00	259340,00	509,25	1355600,00	1164,30
165	2008/12/11 14:15:52	0,00	328,00	1640,00	188880,00	434,60	1351760,00	1162,65
166	2008/12/11 14:16:25	0,00	330,00	1650,00	146500,00	382,75	1356740,00	1164,79
167	2008/12/11 14:16:58	0,00	332,00	1660,00	154120,00	392,58	1355280,00	1164,16
168	2008/12/11 14:17:30	0,00	334,00	1670,00	211170,00	459,53	1352550,00	1162,99
169	2008/12/11 14:18:03	0,00	336,00	1680,00	291510,00	539,92	1353580,00	1163,43
170	2008/12/11 14:18:35	0,00	338,00	1690,00	355180,00	595,97	1353110,00	1163,23
171	2008/12/11 14:19:08	0,00	340,00	1700,00	376050,00	613,23	1352260,00	1162,87
172	2008/12/11 14:19:40	0,00	342,00	1710,00	340040,00	583,13	1353290,00	1163,31
173	2008/12/11 14:20:13	0,00	344,00	1720,00	267620,00	517,32	1354510,00	1163,83
174	2008/12/11 14:20:45	0,00	346,00	1730,00	188870,00	434,59	1353090,00	1163,22
175	2008/12/11 14:21:18	0,00	348,00	1740,00	136920,00	370,03	1349770,00	1161,80
176	2008/12/11 14:21:50	0,00	350,00	1750,00	135230,00	367,74	1353790,00	1163,52
177	2008/12/11 14:22:23	0,00	352,00	1760,00	174280,00	417,47	1351110,00	1162,37
178	2008/12/11 14:22:55	0,00	354,00	1770,00	230370,00	479,97	1351280,00	1162,45
179	2008/12/11 14:23:28	0,00	356,00	1780,00	268330,00	518,01	1354270,00	1163,73
180	2008/12/11 14:24:00	0,00	358,00	1790,00	272270,00	521,79	1352420,00	1162,94
181	2008/12/11 14:24:33	0,00	360,00	1800,00	232820,00	482,51	1345970,00	1160,16

This file is automatically generated: 2008/12/11 14:24:33

Example of data file

APPENDIX B -MUELLER MATRIX FORMALISM

The Mueller matrix formalism is based on the assumption that the polarimetric properties of an optical system can be described with a real 4x4 matrix, *M*, designated as Mueller Matrix. The polarization properties of light can be defined with a four element vector, known as Stokes vector $S=(I,Q,U,V)$, and the interaction with a polarimetric element are described by the product between *S* and *M*. Thanks to the assumed linear character of the interaction between the polarized light and polarimetric components, the Mueller matrix of a system, given by the assembly of different elements, is the product of the single matrices of the system in cascade,

$$M' = \prod_{i=1}^{n} M_i$$

starting from the last element.

In the following some relevant Mueller matrix examples are given.

Linear Polarizer with transmission axis aligned with the reference

$$LP(0) = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Liner Retarder with fast axis aligned with the reference

$$LR(\theta = 0, \delta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & \sin\delta \\ 0 & 0 & -\sin\delta & \cos\delta \end{pmatrix}$$

When the retarder is an half-wave plate, we have $\delta = \pi$ and the associated matrix is:

$$HWLR(0, \pi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and in the case of a quart-wave $\delta = \pi/2$:

$$QWLR\,(0, \frac{\pi}{2}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If we rotate the axis of a polarimetric component, its new Mueller matrix can be found with the following considerations. The interaction with an element with the axis rotated by an arbitrary angle, θ, can be expressed by rotating the incoming Stokes vector by an angle -2θ, i.e. twice the same angle in the opposite direction (polarizers and retarders we are dealing with have symmetric properties with period 180°), so we may think at the same process as if a rotated Stokes vector interact with a non-rotated Mueller matrix. Then we must return in the original reference system by rotating the outcoming Stokes vector by an angle 2θ. Equivalently, the Mueller matrix of the rotated element is expressed by:

$$M(\theta) = R(-2\theta) \cdot M(0) \cdot R(2\theta)$$

where

$$R(2\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

By applying this rule we have:

Linear Polarizer with arbitrary transmission axis

$$LP(\alpha)=\frac{1}{2}\begin{pmatrix}1 & \cos 2\alpha & \sin 2\alpha & 0\\ \cos 2\alpha & \cos^2 2\alpha & \sin 2\alpha\cos 2\alpha & 0\\ \sin 2\alpha & \sin 2\alpha\cos 2\alpha & \sin^2 2\alpha & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

Arbitrary Half Wave Retarder

$$HWLR(\Psi)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos 4\Psi & \sin 4\Psi & 0\\ 0 & \sin 4\Psi & -\cos 4\Psi & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

Arbitrary Quarter Wave Retarder

$$QWLR(\Phi)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos^2 2\Phi & \frac{1}{2}\sin 4\Phi & -\sin 2\Phi\\ 0 & \frac{1}{2}\sin 4\Phi & \sin^2 2\Phi & \cos 2\Phi\\ 0 & \sin 2\Phi & -\cos 2\Phi & 0\end{pmatrix}$$

Generic Retarder

$$M=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & C^2+S^2\cos\delta & SC(1-\cos\delta) & -S\sin\delta\\ 0 & SC(1-\cos\delta) & S^2+C^2\cos\delta & C\sin\delta\\ 0 & S\sin\delta & -C\sin\delta & \cos\delta\end{pmatrix}$$

where C=cos(2ϕ), S=sin(2ϕ). The general Mueller matrix of an ideal retardance plate, with fast axis orientation ϕ ,and retardance δ describes a LCVR.

APPENDIX C – RELATIONSHIPS BETWEEN THE MATRIX ELEMENTS OF THE SAMPLE AND THE FOURIER COEFFICIENTS OF THE MODFULATED SIGNAL

The modulated signal arriving at the MSP detector can be expressed in a Fourier series as:

$$s_{out}(q)=I_q=\frac{a_0}{2}+\sum_{n=1}^{12}\left[a_n\cos(2n\gamma q)+b_n\sin(2n\gamma q)\right]$$

The relationships between the Fourier coefficients and the Mueller matrix elements of the sample are (having some minor typographical errors corrected):

$$a_0 = \frac{1}{2}m_{11} + \frac{1}{4}\beta_3 m_{12} + \frac{1}{4}\beta_4 \cos(2\varepsilon_5)m_{21} + \frac{1}{8}\beta_3\beta_4 \cos(2\varepsilon_5)m_{22} +$$
$$+ \frac{1}{4}\beta_4 \sin(2\varepsilon_5)m_{31} + \frac{1}{8}\beta_3\beta_4 \sin(2\varepsilon_5)m_{32}$$
$$a_1 = \frac{1}{2}\sin\delta_1 \sin(2\varepsilon_3)m_{14} + \frac{1}{4}\sin\delta_1\beta_4 \sin(2\varepsilon_3)\cos(2\varepsilon_3)m_{24} +$$
$$+ \frac{1}{4}\beta_4 \sin\delta_1 \sin(2\varepsilon_3)\sin(2\varepsilon_5)m_{34}$$
$$a_2 = \frac{1}{4}\beta_1 \cos(4\varepsilon_3)m_{12} + \frac{1}{4}\beta_1 \sin(4\varepsilon_3)m_{13} + \frac{1}{8}\beta_1\beta_4 \cos(4\varepsilon_3)\cos(2\varepsilon_5)m_{22} +$$
$$+ \frac{1}{8}\beta_1\beta_4 \sin(4\varepsilon_3)\cos(2\varepsilon_5)m_{23} + \frac{1}{8}\beta_1\beta_4 \cos(4\varepsilon_3)\sin(2\varepsilon_5)m_{32} +$$
$$+ \frac{1}{8}\beta_1\beta_4 \sin(4\varepsilon_3)\sin(2\varepsilon_5)m_{33}$$
$$a_3 = -\frac{1}{8}\beta_1 \sin\delta_2 \sin\alpha_3 m_{42} - \frac{1}{8}\beta_1 \sin\delta_2 \cos\alpha_3 m_{43}$$
$$a_4 = -\frac{1}{4}\sin\delta_1 \sin\delta_2 \cos\alpha_1 m_{44}$$
$$a_5 = \frac{1}{2}\sin\delta_2 \sin\alpha_5 m_{41} + \frac{1}{4}\beta_3 \sin\delta_2 \sin\alpha_5 m_{42}$$
$$a_6 = \frac{1}{4}\sin\delta_1 \sin\delta_2 \cos\alpha_2 m_{44}$$
$$a_7 = -\frac{1}{8}\beta_1 \sin\delta_2 \sin\alpha_4 m_{42} + \frac{1}{8}\beta_1 \sin\delta_2 \cos\alpha_4 m_{43}$$
$$a_8 = \frac{1}{16}\beta_1\beta_2 \cos\alpha_9 (m_{22} + m_{33}) + \frac{1}{16}\beta_1\beta_2 \sin\alpha_9 (m_{32} - m_{23})$$
$$a_9 = \frac{1}{8}\beta_2 \sin\delta_1 \sin\alpha_6 m_{24} + \frac{1}{8}\beta_2 \sin\delta_1 \cos\alpha_6 m_{34}$$
$$a_{10} = \frac{1}{4}\beta_2 \cos\alpha_{11} m_{21} + \frac{1}{8}\beta_2\beta_3 \cos\alpha_{11} m_{22} + \frac{1}{4}\beta_2 \sin\alpha_{11} m_{31} + \frac{1}{8}\beta_2\beta_3 \sin\alpha_{11} m_{32}$$
$$a_{11} = -\frac{1}{8}\beta_2 \sin\delta_1 \sin\alpha_7 m_{24} - \frac{1}{8}\beta_2 \sin\delta_1 \cos\alpha_7 m_{34}$$
$$a_{12} = \frac{1}{16}\beta_1\beta_2 \cos\alpha_{10} (m_{22} - m_{33}) + \frac{1}{16}\beta_1\beta_2 c \sin\alpha_{10} (m_{23} + m_{32})$$
$$b_0 = 0$$
$$b_1 = \frac{1}{2}\sin\delta_1 \cos(2\varepsilon_3)m_{14} + \frac{1}{4}\beta_4 \sin\delta_1 \cos(2\varepsilon_3)\cos(2\varepsilon_5)m_{24} +$$
$$+ \frac{1}{4}\beta_4 \sin\delta_1 \cos(2\varepsilon_3)\sin(2\varepsilon_5)m_{34}$$

$$b_2 = -\frac{1}{4}\beta_1 \sin(4\varepsilon_3)m_{12} + \frac{1}{4}\beta_1 \cos(4\varepsilon_3)m_{13} + \frac{1}{8}\beta_1\beta_4 \cos(4\varepsilon_3)\cos(2\varepsilon_5)m_{23} +$$
$$-\frac{1}{4}\beta_1\beta_4 \sin(4\varepsilon_3)\cos(2\varepsilon_5)m_{22} + \frac{1}{8}\beta_1\beta_4 \cos(4\varepsilon_3)\sin(2\varepsilon_5)m_{33} +$$
$$-\frac{1}{8}\beta_1\beta_4 \sin(4\varepsilon_3)\sin(2\varepsilon_5)m_{32}$$
$$b_3 = -\frac{1}{8}\beta_1 \sin\delta_2 \cos\alpha_3 m_{43} + \frac{1}{8}\beta_1 \sin\delta_2 \sin\alpha_3 m_{43}$$

$$b_4 = \frac{1}{4}\sin\delta_1 \sin\delta_2 \sin\alpha_1 m_{44}$$
$$b_5 = -\frac{1}{2}\sin\delta_2 \cos\alpha_5 m_{41} - \frac{1}{4}\beta_3 \sin\delta_2 \cos\alpha_5 m_{42}$$
$$b_6 = -\frac{1}{4}\sin\delta_1 \sin\delta_2 \sin\alpha_2 m_{44}$$
$$b_7 = -\frac{1}{8}\beta_1 \sin\delta_2 \cos\alpha_4 m_{42} - \frac{1}{8}\beta_1 \sin\delta_2 \sin\alpha_4 m_{43}$$
$$b_8 = -\frac{1}{16}\beta_1\beta_2 \sin\alpha_9 (m_{22} + m_{33}) - \frac{1}{16}\beta_1\beta_2 \cos\alpha_9 (m_{23} - m_{32})$$
$$b_9 = -\frac{1}{8}\beta_2 \sin\delta_1 \cos\alpha_6 m_{24} + \frac{1}{8}\beta_2 \sin\delta_1 \sin\alpha_6 m_{34}$$
$$b_{10} = -\frac{1}{4}\beta_2 \sin\alpha_{11} m_{21} - \frac{1}{8}\beta_2\beta_3 \sin\alpha_{11} m_{22} + \frac{1}{4}\beta_2 \cos\alpha_{11} m_{31} + \frac{1}{8}\beta_2\beta_3 \cos\alpha_{11} m_{32}$$
$$b_{11} = \frac{1}{8}\beta_2 \sin\delta_1 \cos\alpha_7 m_{24} - \frac{1}{8}\beta_2 \sin\delta_1 \sin\alpha_7 m_{34}$$
$$b_{12} = -\frac{1}{16}\beta_1\beta_2 \sin\alpha_{10} (m_{22} - m_{33}) + \frac{1}{16}\beta_1\beta_2 \cos\alpha_{10} (m_{32} + m_{23})$$

Mueller matrix elements

$$m_{44} = \frac{4}{\sin\delta_1 \sin\delta_2}\left(-\frac{a_4}{\cos\alpha_1} + \frac{a_6}{\cos\alpha_2}\right)$$
$$m_{43} = 8\frac{-a_3 \cos\alpha_3 + b_3 \sin\alpha_3 + a_7 \cos\alpha_4 - b_7 \sin\alpha_4}{\beta_1 \sin\delta_2}$$
$$m_{42} = -8\frac{a_3 \sin\alpha_3 + b_3 \cos\alpha_3 + a_7 \sin\alpha_4 + b_7 \cos\alpha_4}{\beta_1 \sin\delta_2}$$
$$m_{41} = \frac{-\beta_3 m_{42}}{2} - \frac{4b_5}{\cos\alpha_5 \sin\delta_2}$$
$$m_{24} = 8\frac{a_9 \sin\alpha_6 - b_9 \cos\alpha_6 - a_{11} \sin\alpha_7 + b_{11} \cos\alpha_7}{\beta_2 \sin\delta_1}$$

$$m_{34} = 8\frac{a_9\cos\alpha_6 + b_9\sin\alpha_6 - a_{11}\cos\alpha_7 - b_{11}\sin\alpha_7}{\beta_2\sin\delta_1}$$

$$m_{14} = \frac{-\beta_4\cos(2\varepsilon_5)m_{24}}{2} + \frac{4b_1}{\cos(2\varepsilon_3)\sin\delta_1} - \frac{\beta_4\sin(2\varepsilon_5)m_{34}}{2}$$

$$m_{22} = 16\frac{a_8\cos\alpha_9 + a_{12}\cos\alpha_{10} - b_8\sin\alpha_9 - b_{12}\sin\alpha_{10}}{\beta_1\beta_2}$$

$$m_{33} = 16\frac{a_8\cos\alpha_9 - a_{12}\cos\alpha_{10} - b_8\sin\alpha_9 + b_{12}\sin\alpha_{10}}{\beta_1\beta_2}$$

$$m_{23} = 16\frac{-a_8\sin\alpha_9 + a_{12}\sin\alpha_{10} - b_8\cos\alpha_9 + b_{12}\cos\alpha_{10}}{\beta_1\beta_2}$$

$$m_{32} = 16\frac{a_8\sin\alpha_9 + a_{12}\sin\alpha_{10} + b_8\cos\alpha_9 + b_{12}\cos\alpha_{10}}{\beta_1\beta_2}$$

$$m_{12} = \frac{16a_2\cos(4\varepsilon_3) - 16b_2\sin(4\varepsilon_3) - \beta_1\beta_4\cos(2\varepsilon_5)m_{22} - \beta_1\beta_4\sin(2\varepsilon_5)m_{32}}{2\beta_1}$$

$$m_{13} = \frac{16a_2\sin(4\varepsilon_3) + 16b_2\cos(4\varepsilon_3) - \beta_1\beta_4\cos(2\varepsilon_5)m_{23} - \beta_1\beta_4\sin(2\varepsilon_5)m_{33}}{2\beta_1}$$

$$m_{21} = \frac{16a_{10}\cos\alpha_{11} - 16b_{10}\sin\alpha_{11} - \beta_2\beta_3 m_{22}}{2\beta_2}$$

$$m_{31} = \frac{-(\beta_2\beta_3 m_{32} - 16b_{10}\cos\alpha_{11} - 16a_{10}\sin\alpha_{11})}{2\beta_2}$$

$$m_{11} = 4a_0 - \frac{1}{2}\beta_3 m_{12} - \frac{1}{2}\beta_4\cos(2\varepsilon_5)m_{21} - \frac{1}{4}\beta_3\beta_4\cos(2\varepsilon_5)m_{22} +$$

$$-\frac{1}{2}\beta_4\sin(2\varepsilon_5)m_{31} - \frac{1}{4}\beta_3\beta_4\sin(2\varepsilon_5)m_{32}$$

The MSP calibration is done by performing a series of measurements without sample. Owing the Mueller matrix of air to be the unit 4x4 matrix, it is possibile to obtain relations between the Fourier coefficients from the signal analysis and the alignment and retardance error in the setup:

$$\varepsilon_3 = \frac{1}{4}\arctan\left(\frac{b_8}{a_8}\right) - \frac{1}{4}\arctan\left(\frac{b_{10}}{a_{10}}\right)$$

$$\varepsilon_4 = \frac{1}{2}\arctan\left(\frac{b_2}{a_2}\right) - \frac{1}{2}\arctan\left(\frac{b_6}{a_6}\right) + \frac{1}{4}\arctan\left(\frac{b_8}{a_8}\right) - \frac{1}{4}\arctan\left(\frac{b_{10}}{a_{10}}\right)$$

$$\varepsilon_5 = \frac{1}{2}\arctan\left(\frac{b_2}{a_2}\right) + \frac{1}{2}\arctan\left(\frac{b_8}{a_8}\right) - \frac{1}{2}\arctan\left(\frac{b_{10}}{a_{10}}\right)$$

$$\delta_1 = \arccos\left(\frac{a_{10}\cos\alpha_9 - a_8\cos\alpha_{11}}{a_{10}\cos\alpha_9 + a_8\cos\alpha_{11}}\right)$$

$$\delta_2 = \arccos\left[\frac{a_2\cos\alpha_9 - a_8\cos(4\varepsilon_3 - 2\varepsilon_5)}{a_2\cos\alpha_9 + a_8\cos(4\varepsilon_3 - 2\varepsilon_5)}\right]$$

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