E-KPol Temperature Calibration

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1. Introduction

This technical report describes the temperature calibration of the E-KPol, a LCVR-based polarimeter for solar K-corona observation during total solar eclipses (fig.1) [1].

Figure 1 – E-KPol polarimeter.

1.1. E-KPol

The E-KPol polarimeter assembly is based on a Liquid Crystal Variable Retarder, LCVR, where the retardance is modified by varying an applied voltage. A fixed achromatic Quarter Wave plate is mounted with its fast axis forming a 45[deg] angle with the LCVR fast axis, in a rotator configuration: the polarization axis of the incoming linearly polarized coronal radiation is rotated by half of the LCVR retardance. After that, a fixed Linear Polarizer acts as an analyzer (fig.2).

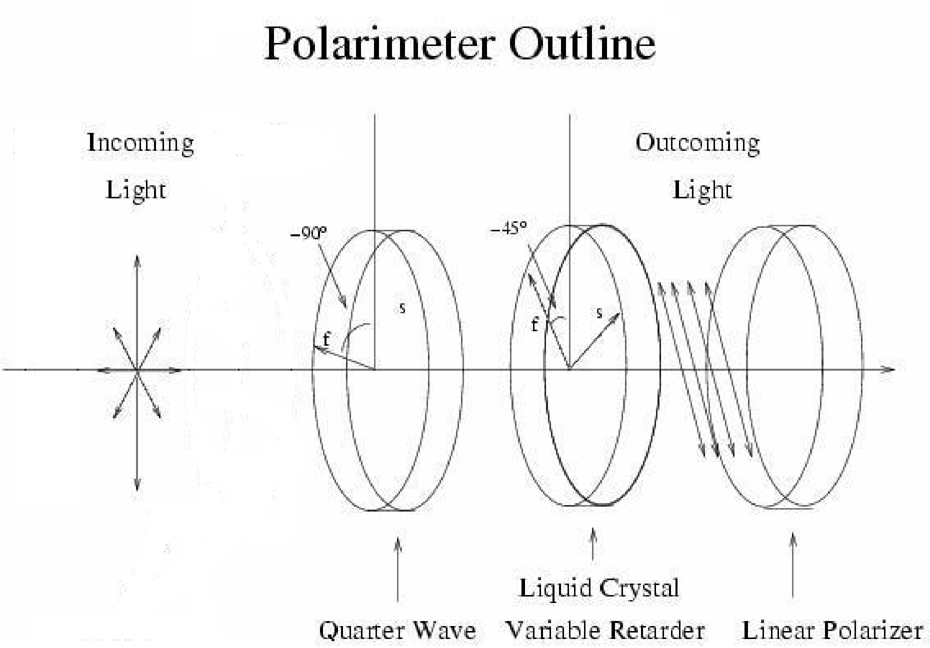


Figure 2 - E-KPol polarimetric assembly, scheme of components.

A polarimeter like E-KPol, called KPol, is used in spatial mission (HERSCHEL/SCORE). Schematic view of this instrument and of KPol are in figure 3.

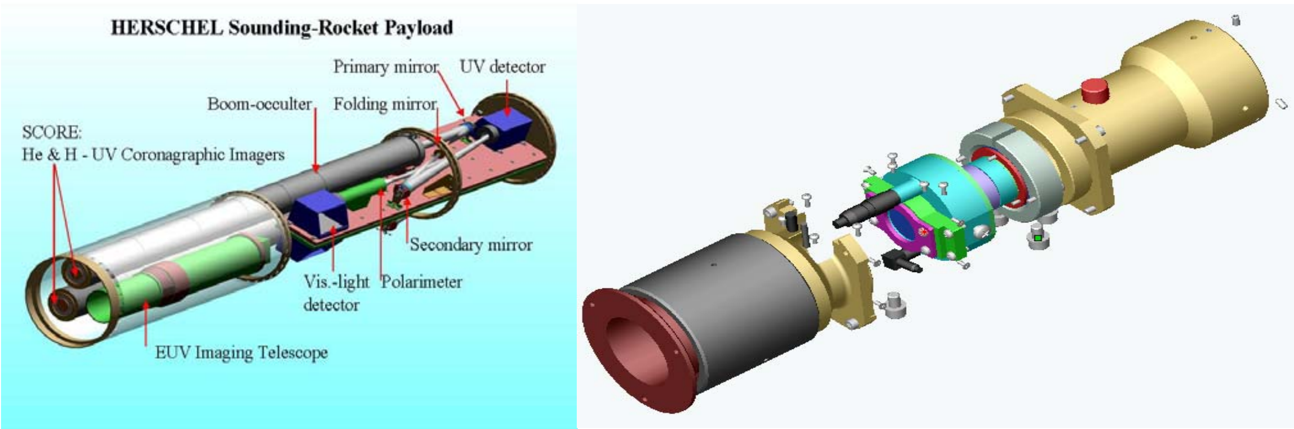


Figure 3 - HERSCHEL instrument and KPol schematic assembly.

The main purpose of the calibration activity is to characterize the polarimeter response, i.e. the rotation angle, at different voltages and temperatures.

1.2. LCVR

The LCVR is a commercial Liquid Crystal Variable Retarder manufactured by Meadowlark Optics [2]. A picture of this device is in fig.4. It consists of two windows separated by few microns. The space between the windows is filled by nematic liquid crystals. Thanks to a thin ITO (Indium Tin Oxide) layer the windows work as electrodes. So a voltage can be applied.

Figure 4 – LCVR .

Picture extracted from Meadowlark Optics 2007-2008 Catalogue

This device, liquid crystal based, change birefringence (than retardance) in function of applied voltages. For this property is used in aerospatial applications (have no moving parts).

The liquid crystal consists of elongated molecules and the retardance is given by their orientation. If no voltage is applied the molecules lie parallel to the LCVR windows and there's no retardance. If there's a voltage is applied between the windows, the molecules orient their main axes parallel to the electrical field and so the retardance starts to decrease (fig.5).

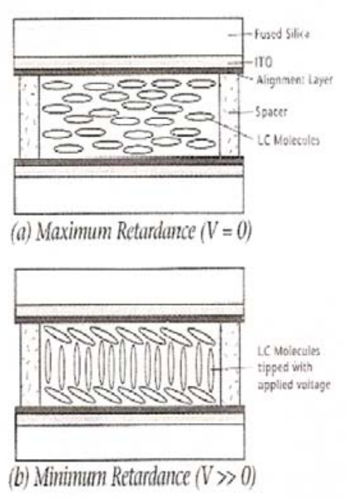


Figure 5 – LCVR behaviour.
Picture extracted from Meadowlark Optics 2007-2008 Catalogue

LCVRs mount a heater and a temperature sensor. The LCVR response as a function of temperature has been calibrated in the laboratory. The calibration has been performed temperatures for three temperatures: 23°C, 30°C and 34°C. Since the LCVR does not have a cooler, the minimum operational temperature is room temperature (approximately 22°C). The specifications of the tested LCVR are summarized in tab.1.

Device	Liquid crystal variable retarder
Manufacturer	Meadowlark Optics Inc.
Model	LRC-200
S/N	04-579
Outside Diameter	2.00 in
Clear Aperture	0.70 in
Substrate Material	Synthetic fused silica
Retarder Material	Nematic liquid crystal
Compensator Material	Birefringent polymer
Retardance wavelenght	0 to 3/4 wave
Operating wavelength	400 to 700 nm
Temperature range	0-50 °C

Tab. 1 – LCVR properties.

2. Setup

Figs. 6 and 7 show a schematic layout of the laboratory setup .

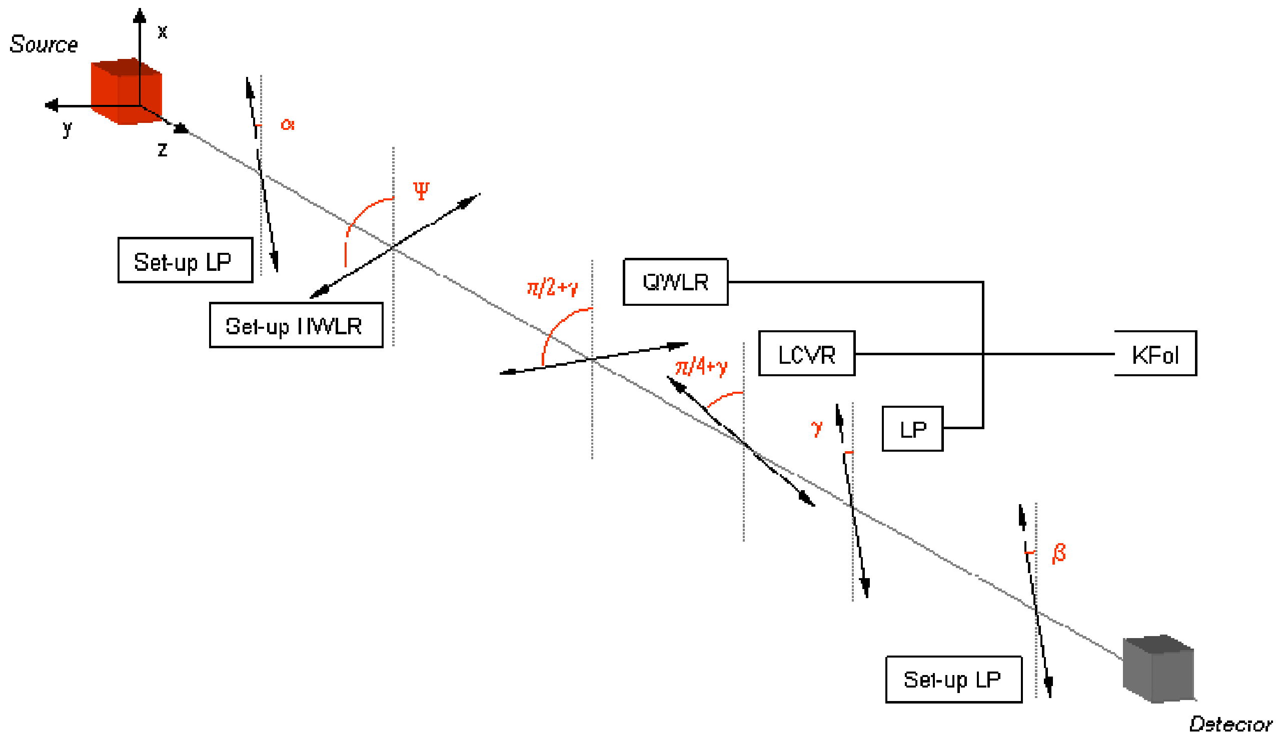


Figure 6 - Schematic layout of the set-up for temperature calibration.

The source is a halogen lamp feeding the entrance slit of a monochromator that selects the wavelength of the output radiation. A fiber optic bundle from the exit slit is collimated to illuminate the polarimetric components of the set-up: the linear pre-polarizer, the half wave plate (HWLR), E-KPol and finally the linear analyzer, in front of the photomultiplier tube (PMT) detector. The transmission axis of the linear pre-polarizer defines the polarization direction of the input radiation. The input linear polarization is then rotated by the half wave plate, and modulated by the E-KPol.

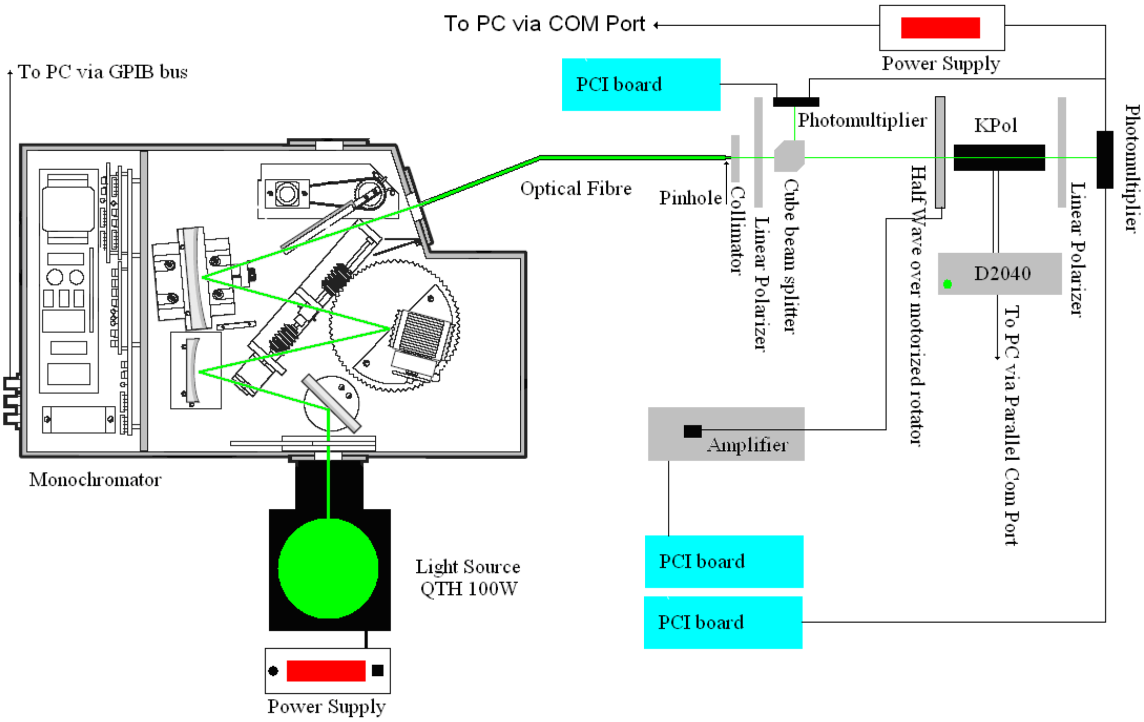


Figure 7 - Schematic laboratory set-up for E-KPol temperature calibration.

Owing to the low light levels, a PMT has been used as a detector. The stability of the source is monitored by a second detection channel, in which a photodiode is fed by a beam splitter positioned behind the linear pre-polarizer. Other details are reported in [3].

3. Setup alignment

By sending an unpolarized light beam, the signal recorded by a detector placed behind the set-up is:

$$Signal \equiv S = \begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix} \cdot LP(\beta) \cdot Kpol(\gamma,\delta) \cdot HWLR(\Psi) \cdot LP(\alpha) \cdot \begin{pmatrix}1\\0\\0\\0\end{pmatrix} =$$

$$= \frac{1}{8}\{1+\cos[2(\gamma-\beta)]\}\{1+\cos\{2[(2\Psi+\rho)-(\alpha+\gamma)]\}\} \qquad (3.1)$$

where $\rho = \frac{\delta}{2}$.

Maximizing the signal, we measure:

$$\begin{cases} \alpha + \beta = 2\psi_0 \\ \gamma - \beta = \chi_0 \end{cases} \Rightarrow 2\psi_0 + \chi_0 = (\alpha + \gamma) = \phi_0 \qquad \textbf{(3.2)}$$

From set-up calibration we obtain:

$$\phi_0 = \chi_0 + 2\psi_0 = -2.4\deg.$$

Details of set-up calibration are given in ref.[3].

4. Data acquisition and data analysis

The bandwidth of E-KPol color filter is centred at 620nm. Setting the monochromator at 0^{th} order,the bandpass is defined by the polarimeter color filter.

4.1. Rotation as a function of voltage

The EKPol output as a function of the set-up half wave plate rotation has been measured at three different LCVR temperatures (23, 30 and 34° C). Each measurement has been repeated at LCVR different voltages. In this way, for each voltage, a Malus curve has been obtained. Fig. 8 gives an example of such curves.

T_{LCVR} = 23 °C

V_{LCVR} = 2500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposure time = 10 s

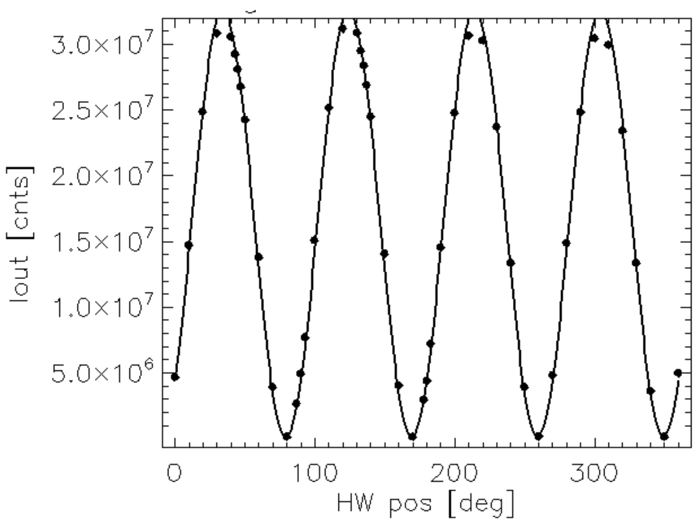


Figure 8 - Example of Malus curve.

The trigonometric function used to fit the data is:

$$y = a \cdot \cos\left[\left(\frac{\pi}{180}\right) \cdot 2 \cdot (2x + 30 + b) + 1\right] + c \qquad \textbf{(4.1)}$$

In the case of the data set plotted in fig. 8, the fit parameters obtained are:

a = (-1.62 ± 0.02)×10^7

b = 322.61 ± 0.20

c = (16.37 ± 0.12)×10^6

The phase "b" yields the value of polarimetric rotation. In this case:

Φ = b - 30° = (292.61 ± 0.20)°

We estimate also the modulation factor defined as:

$$\mu = \frac{\boldsymbol{S}_{\max} - \boldsymbol{S}_{\min}}{\boldsymbol{S}_{\max} + \boldsymbol{S}_{\min}} = 0.99 \qquad \textbf{(4.2)}$$

The Malus curves obtained for all the acquired data set are reported in Appendix A.

The results are summarized in tabs. 2,3 and 4.

T=23°C				
V [mV]	**Rot [deg]**	**Err.Rot [deg]**	μ	**Date**
0	324,8	0,2	0,96	16/05/2008
1000	324,5	0,2	0,96	16/05/2008
1500	320,7	0,5	0,95	23/05/2008
2000	319,5	0,2	0,95	16/05/2008
2500	292,6	0,2	0,99	16/05/2008
3000	255,6	0,2	0,99	16/05/2008
3500	218,3	0,2	0,97	16/05/2008
3750	186,1	0,2	1,00	23/05/2008
4000	184,9	0,2	0,99	16/05/2008
4500	147,0	0,3	0,99	13/05/2008
5000	130,8	0,2	0,99	16/05/2008
5400	110,4	0,2	1,00	13/05/2008
6000	93,1	0,2	1,00	16/05/2008
7000	66,4	0,2	0,98	13/05/2008
8000	49,8	0,2	0,99	16/05/2008
9000	32,3	0,2	0,99	23/05/2008
10000	27,5	0,2	0,98	13/05/2008
11000	17,0	0,2	0,97	23/05/2008
12000	11,9	0,2	1,00	23/05/2008

Tab. 2 – Rotation vs. Voltage for T=23°C.

T=30°C				
V [mV]	Rot [deg]	Err.Rot [deg]	μ	Date
0	315,2	0,2	0,98	21/05/2008
1000	315,0	0,2	0,84	22/05/2008
2000	305,3	0,2	0,97	21/05/2008
2500	274,6	0,2	0,84	22/05/2008
3000	233,3	0,2	0,84	22/05/2008
3500	197,3	0,3	0,83	22/05/2008
4000	156,9	0,2	0,98	21/05/2008
4500	137,0	0,3	0,97	22/05/2008
5000	114,1	0,2	0,99	22/05/2008
5400	98,5	0,2	1,00	22/05/2008
6000	74,7	0,2	0,99	21/05/2008
7000	56,5	0,2	0,98	22/05/2008
8000	39,5	0,2	0,95	21/05/2008
9000	30,1	0,2	0,99	22/05/2008
10000	20,7	0,2	0,94	21/05/2008

Tab. 3 – Rotation vs. Voltage for T=30°C.

T=34°C				
V [mV]	Rot [deg]	Err.Rot [deg]	μ	Date
0	309,2	0,2	0,99	30/05/2008
1000	309,0	0,2	0,99	29/05/2008
2000	300,5	0,2	0,94	30/05/2008
2500	264,6	0,2	0,99	29/05/2008
3000	226,0	0,2	0,98	29/05/2008
3500	187,9	0,3	0,99	29/05/2008
4000	152,4	0,2	0,99	30/05/2008
4500	129,5	0,3	0,99	29/05/2008
5000	105,7	0,2	0,98	29/05/2008
5400	88,9	0,2	1,00	29/05/2008
6000	71,7	0,2	1,00	30/05/2008
7000	50,3	0,2	0,99	29/05/2008
8000	37,3	0,2	0,97	30/05/2008
9000	26,7	0,2	0,98	29/05/2008
10000	19,5	0,2	0,96	30/05/2008

Tab. 4 – Rotation vs. Voltage for T=34°C.

For each temperature, the LCVR Rotation vs. Voltage curves are shown in figures 9-10-11.

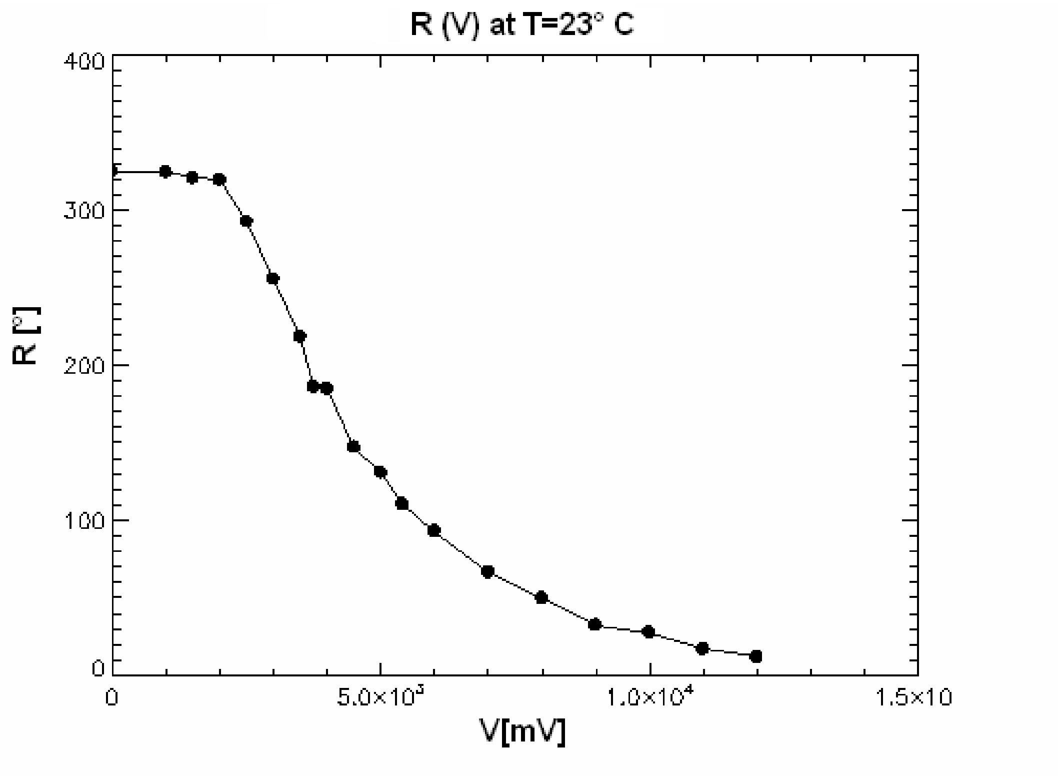


Figure 9 - Rotation vs. applied voltage at T =23° C

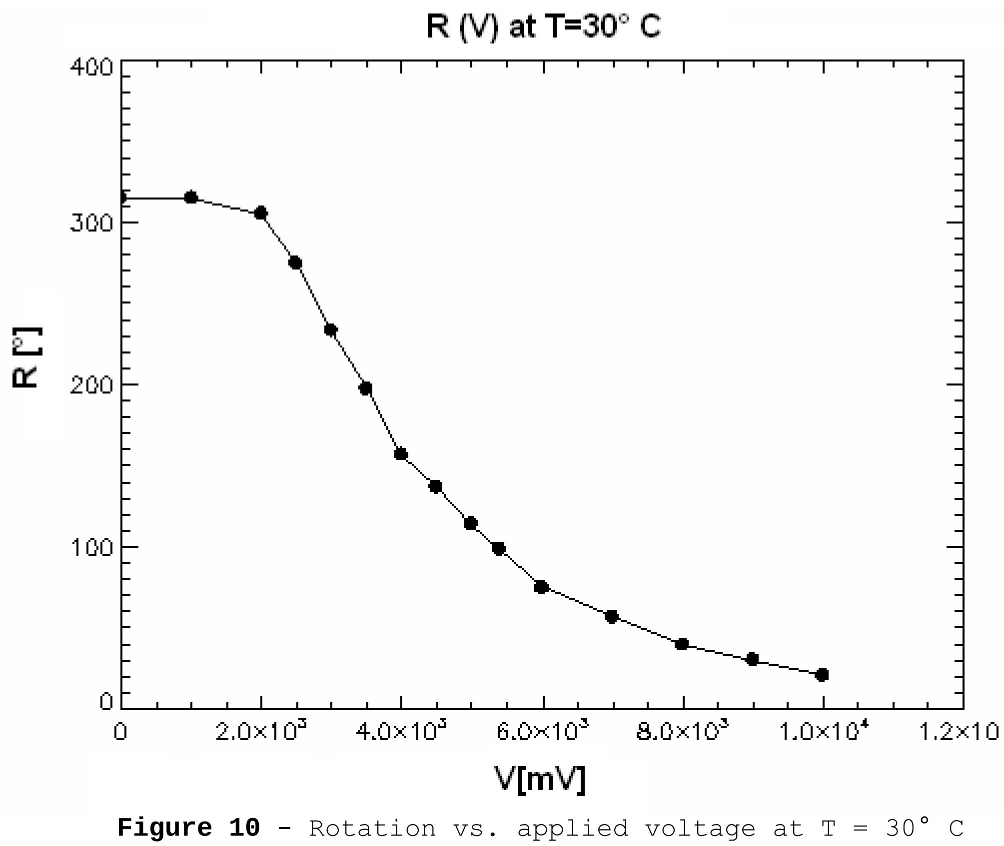


Figure 10 - Rotation vs. applied voltage at T = 30° C

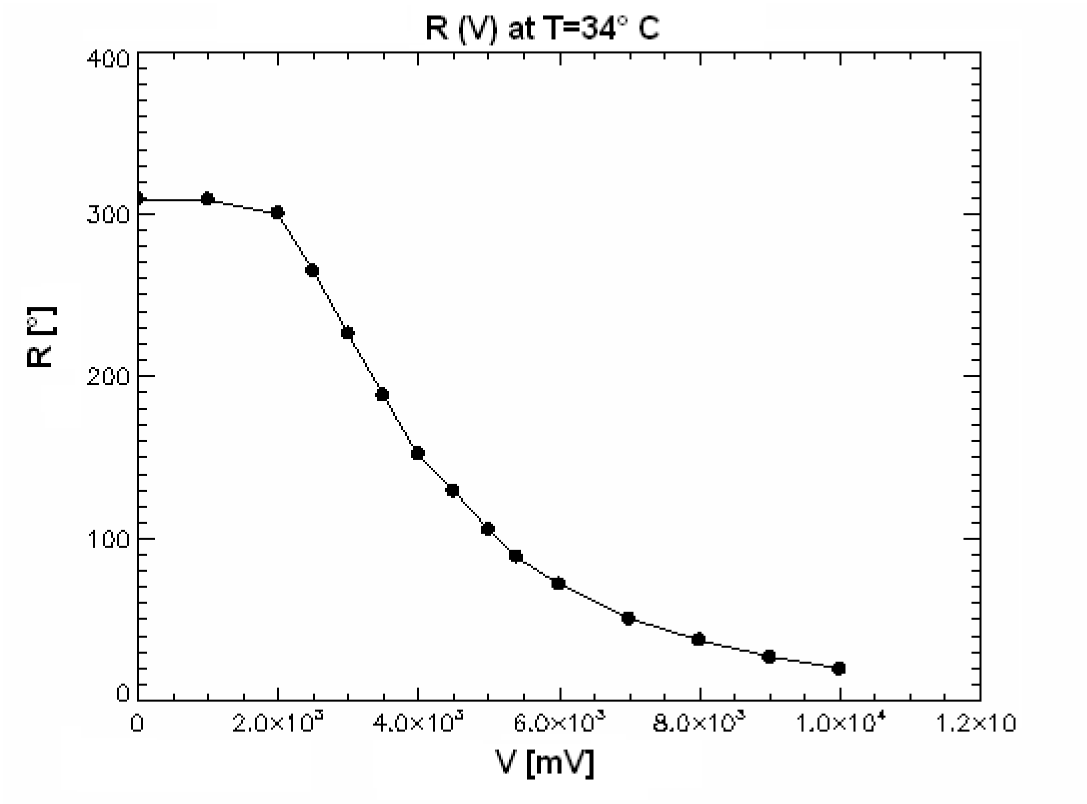


Figure 11 - Rotation vs. applied voltage at T =34° C

The tree previous curves are over-plot for comparison in fig. 12.

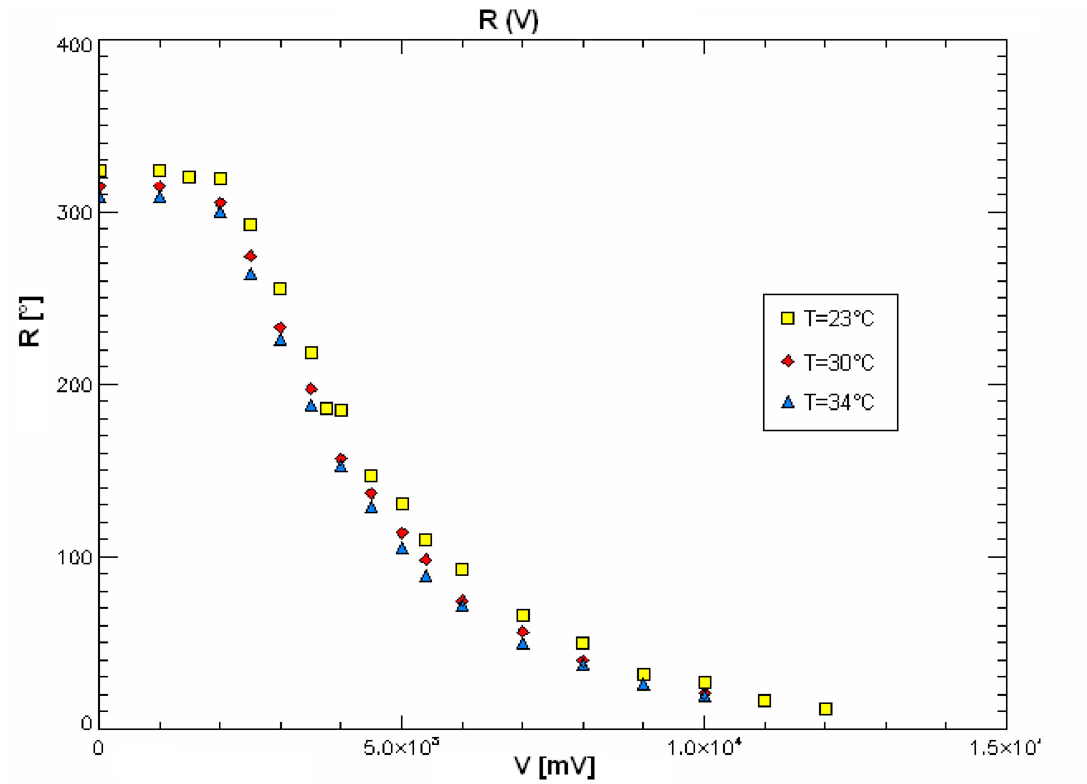


Figure 12 - Rotation vs. applied voltage for different temperatures.

The rotation vs. voltage profile has a well defined physical explanation. It is the result of the molecules elastic deformation in a liquid crystal. If an electric field is applied across a liquid crystal, the field strain exerts a stress on the crystal's molecules similar to that qualitatively described in fig. 13.

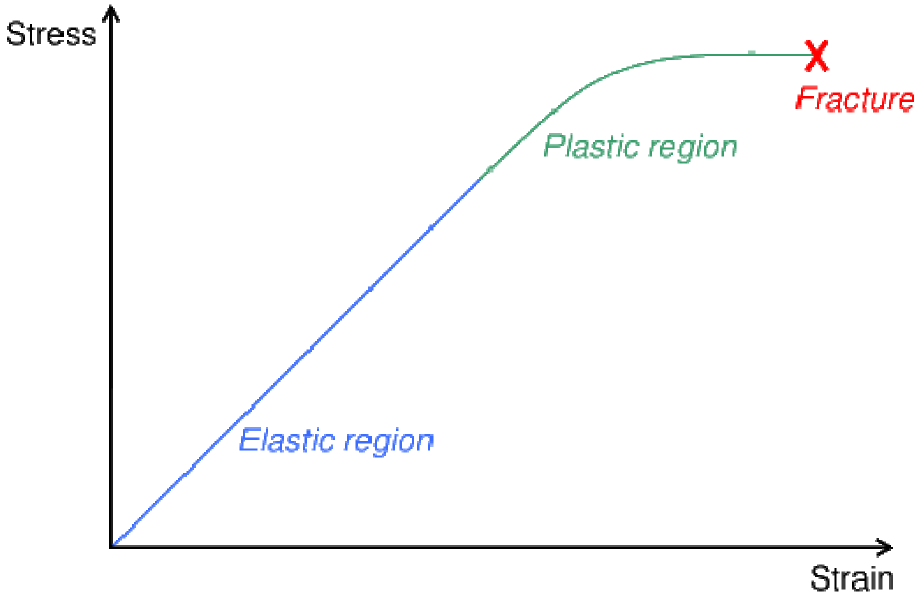


Figure 13 - Diagram of a Stress-strain curve, showing the relationship between stress (force applied) and strain (deformation).

The crystal's molecules stress can be described by a logistic curve (fig. 14).

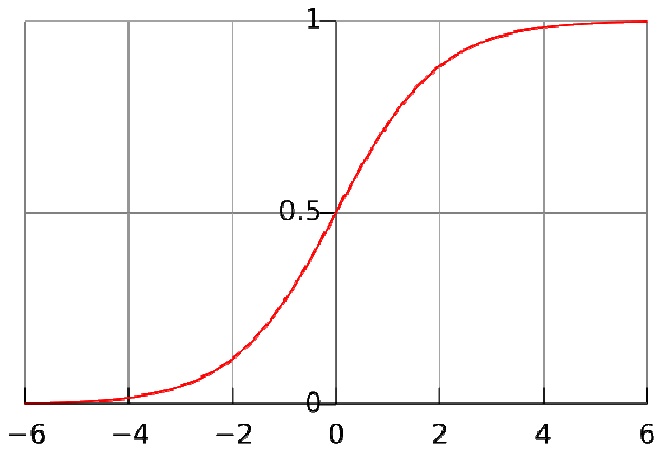


Figure 14 - Standard logistic sigmoid function.

A logistic function or logistic curve is the most common sigmoid curve. A simple logistic function may be defined by the mathematical formula:

$$P(t) = \frac{1}{1+e^{-t}} \quad \textbf{(4.3)}$$

This logistic function is the solution of the simple first-order non-linear differential equation:

$$\frac{dP}{dt} = P(1-P) \quad \textbf{(4.4)}$$

For P << 1, it increases like an exponential; for P ~ 1 the derivate tends to 0 and the function tends to a costant value. The region of the experimental logistic curve at high voltages corresponds to the plastic deformation. Figure 13 shows that at higher temperatures the rotation is smaller than at lower temperatures; furthermore there is a larger temperature difference at low voltages: when the voltage increases the three logistic curves tend to the same value.

4.2. Retardance as a function of temperature

According to a semi-empirical formula, retardance and temperature are connected by this relation:

$$\delta(T) = \delta_0 S = \delta_0 \left(1 - \frac{T}{T_C}\right)^{\beta} \quad \textbf{(4.5)}$$

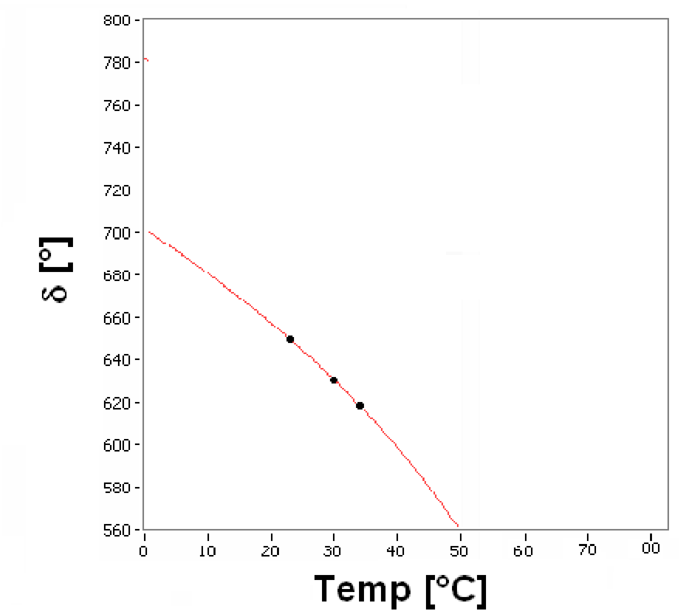
Where S is an order parameter, δ_0 the retardance for S=1, T_C the nematic-isotropic transition temperature (near T_C, S drops noncontinuously to zero) and β a critical exponent related to the phase transition.

Theoretical explanation of this equation is given in Appendix C.

For all different voltages, formula 4.5 is used to fit the experimental values of retardance vs. temperatures. For V=0 V, for example, data are resumed in tab.2. Plot and fit parameters are in fig.15.

Temp [°C]	Ret [°]	Err. Ret.[°]
23	649.6	0.4
30	630.4	0.4
34	618.4	0.4

Tab. 2 – Measured values of retardance with their errors.



$$\begin{cases} \delta_0 = 701 \pm 1.4 \deg \\ T_C = 76.9 \pm 0.2°C \\ \beta = 0.216 \pm 0.004 \end{cases}$$

Figure 15 – Retardance vs. Temperature for V=0V.

For the other fits T_C has been fixed because, for the same liquid crystal in the same conditions, it doesn't change.

All data plots and respectively fit are reported in Appendix B.
All analyzed data are resumed in fig. 16.

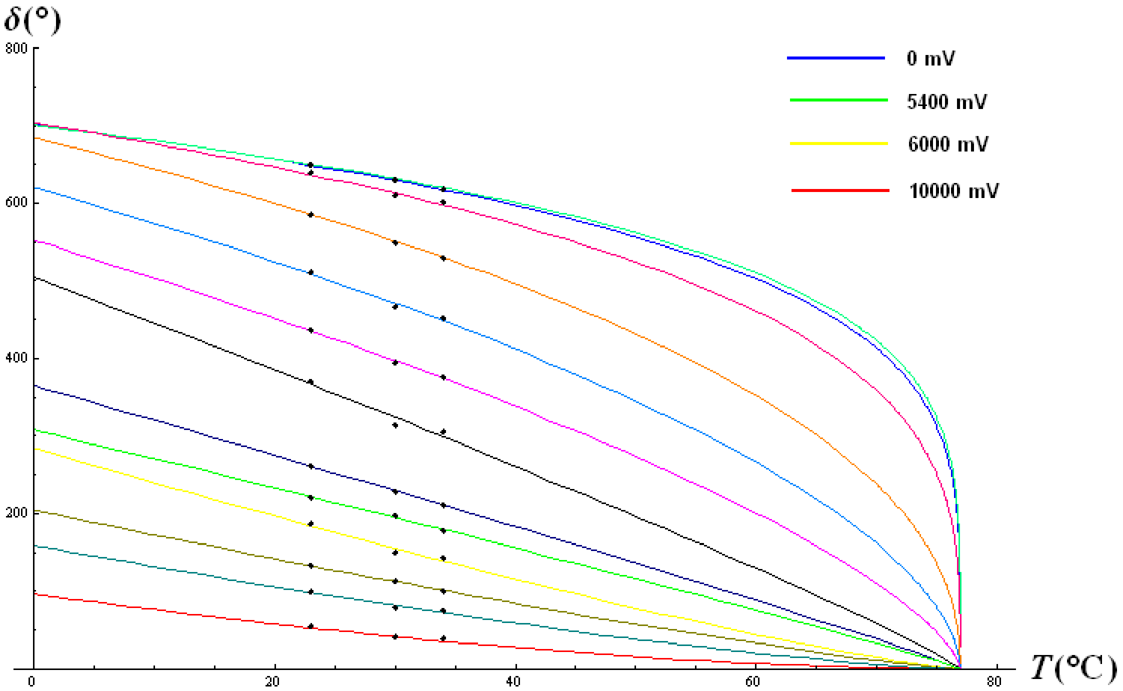


Figure 16 - Rotation in function of temperature at different voltages applied.

Note that for increasing voltages there is a change in the concavity of the curves. This is due to the β factor. From 0 to 5400 mV β is lower than 1; from 6000 to 10000 mV β is higher than 1. Furthermore note that all curves drop to zero at the transition temperature T_C = 76.9° C.
By plotting the retardance derivate versus the temperature as a function of voltage (fig.17), 3 regions can be located in the resulting graph. In the first one (from 0 to 3000 mV), the derivate increases . In the second one (from 3000 to 6000 mV), the derivate has a maximum, and finally in the third one (from 6000 to 10000 mV) the derivate decreases.

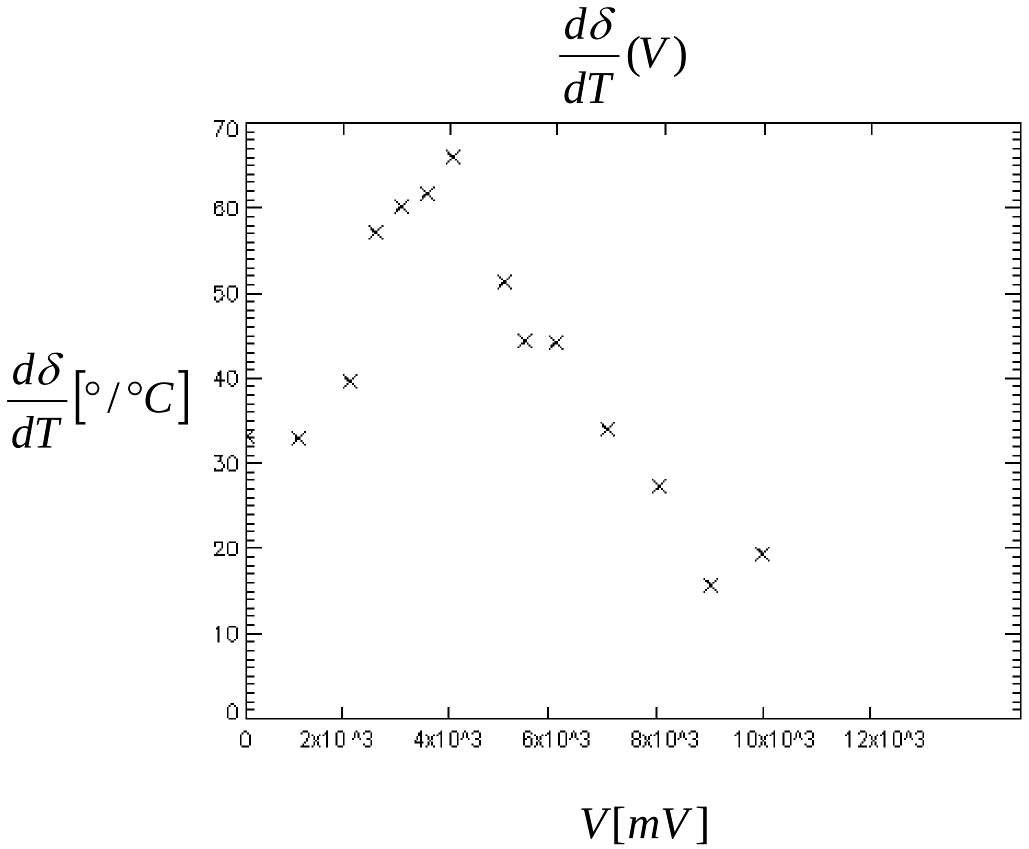


Figure 17 - Retardance derivate respect to the temperature as a function of voltage.

5. Conclusions

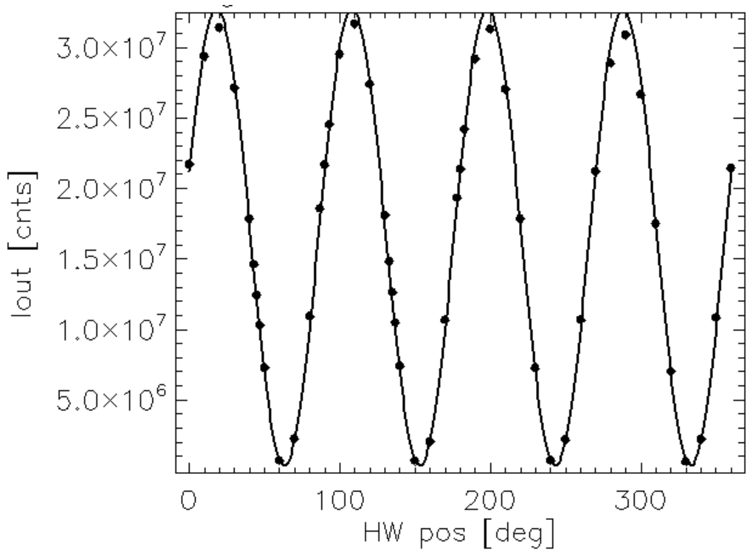
This rate of retardance as a function voltage, shown in Fig. 17, is explanated in the following physical way. For low voltages (0 ÷ 3 V) the molecules orientation is still parallel to the LCVR's windows and the retardance doesn't change with temperature. In this voltage region, the liquid crystal surface tension prevails. For intermediate voltages (3 ÷ 6 V), the electrical field starts to increase but it is not strong enough, yet, to win the molecules' thermal motions. In this voltage range, the retardance variation with temperature is at maximum. For high voltages (> 6 V), the electrical field prevails over the molecules' thermal motions and the retardance is less sensitive to the liquid crystal’s temperature.

In conclusion, the LCVR is less sensitive to the temperature when operated at high voltages ($\geq$ 5-6V).

Appendix A- Rotation vs. Voltage Data Analysis

T_{LCVR} = 23°C

V_{LCVR} = 0 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s

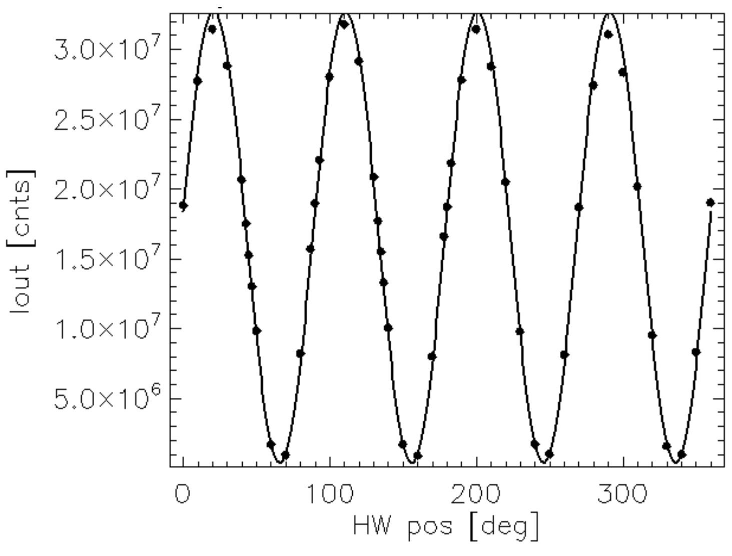


FIT PARAMETERS:
a = −16181345±0.02×10^7
b = 264.75±0.2
c = 16536423±1.4×10^5

PHASE:
Φ = b + 60° = (324.75±0.2)°

MODULATION FACTOR:
μ = 0.96

V_{LCVR} = 1000 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s



FIT PARAMETERS:
a = −16211884±0.02×10^7
b = 264.53±0.2
c = 16573591±1.4×10^5

PHASE:
Φ = b + 60° = (324.53±0.2)°

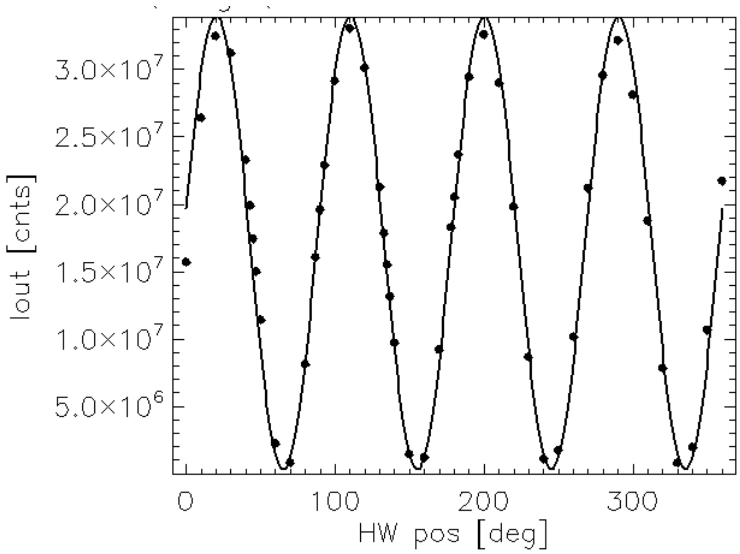
MODULATION FACTOR:
μ = 0.96

V_{LCVR} = 1500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16812590±0.07×10^7

b = 260.68±0.5

c = 17143746±0.4×10^6

PHASE:

Φ = b + 60° = (320.68±0.5)°

MODULATION FACTOR:

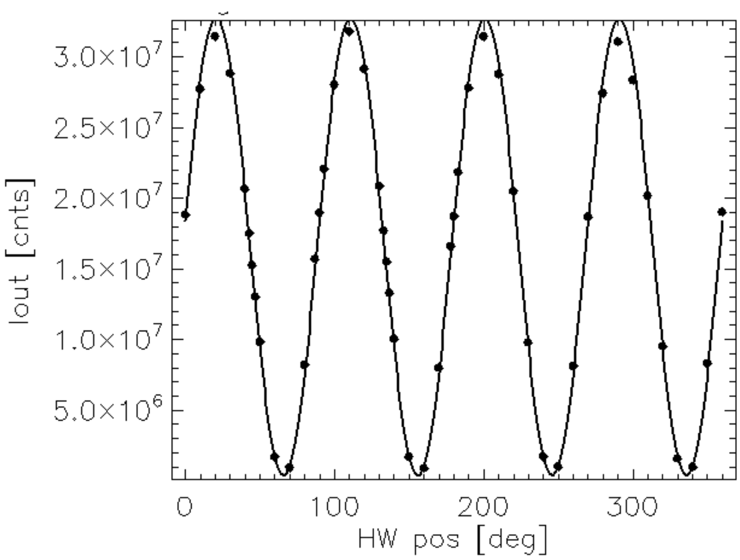
μ = 0.95

V_{LCVR} = 2000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16169545±0.02×10^7

b = 259.54±0.2

c = 16560705±1.4×10^5

PHASE:

Φ = b + 60° = (319.54±0.2)°

MODULATION FACTOR:

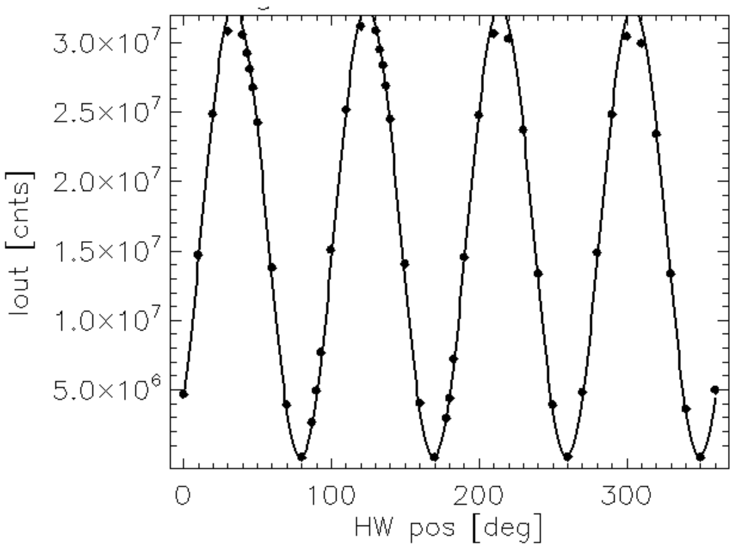
μ = 0.95

V_{LCVR} = 2500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16213757 \pm 0.02\times10^7$

b = 322.61 ± 0.2

c = $16367023 \pm 1.2\times10^5$

PHASE:

Φ = b - 30° = $(292.61 \pm 0.2)°$

MODULATION FACTOR:

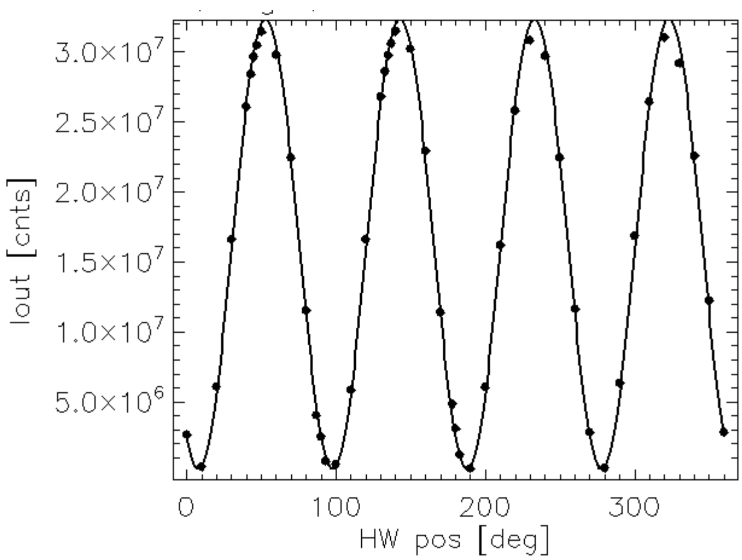
μ = 0.99

V_{LCVR} = 3000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16096393 \pm 0.02\times10^7$

b = 195.58 ± 0.2

c = $16288858 \pm 1.2\times10^5$

PHASE:

Φ = b + 60° = $(255.58 \pm 0.2)°$

MODULATION FACTOR:

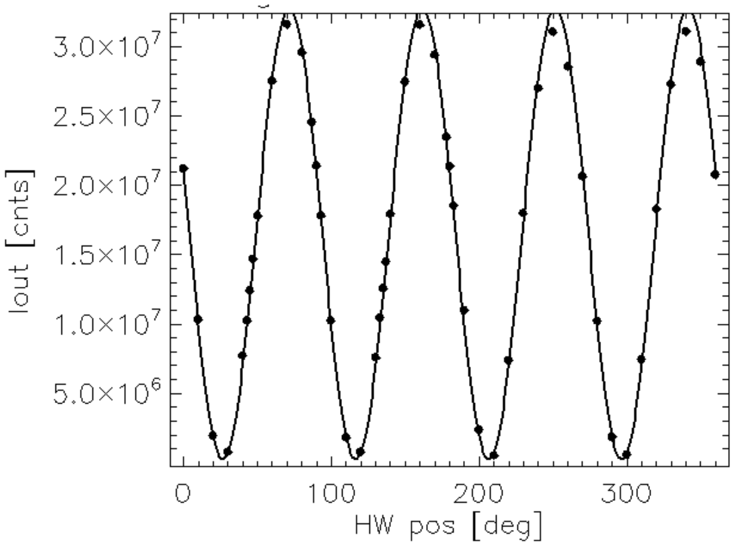
μ = 0.99

V_{LCVR} = 3500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16210703 \pm 0.02\times10^7$

b = 158.35 ± 0.2

c = $16479069 \pm 1.4\times10^5$

PHASE:

Φ = b + 60° = $(218.35 \pm 0.2)°$

MODULATION FACTOR:

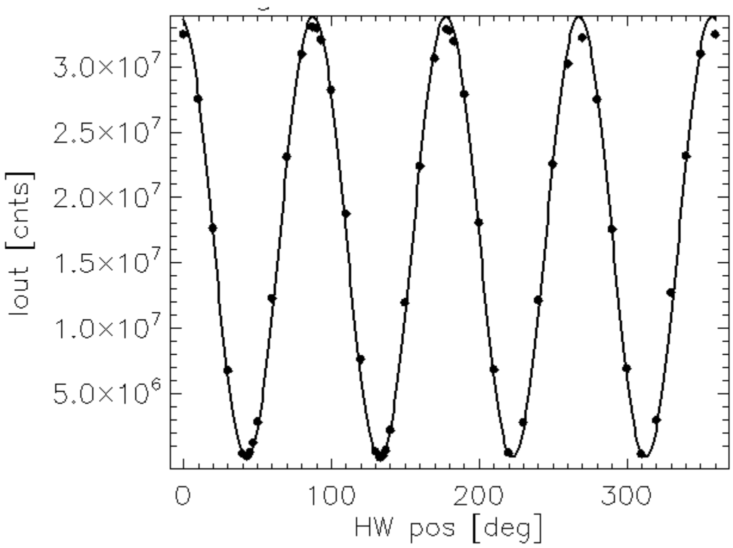
μ = 0.97

V_{LCVR} = 3750 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16840293 \pm 0.02\times10^7$

b = 216.09 ± 0.2

c = $16969605 \pm 0.1\times10^5$

PHASE:

Φ = b - 30° = $(186.09 \pm 0.2)°$

MODULATION FACTOR:

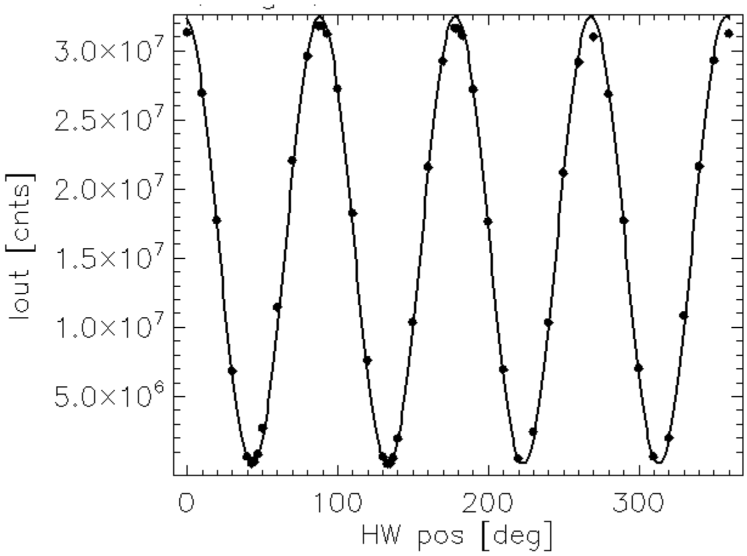
μ = 1.00

V_{LCVR} = 4000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16158254\pm0.02\times10^7$

b = 214.94 ± 0.2

c = $16295080\pm1.2\times10^5$

PHASE:

Φ = b - 30° =

(184.94±0.2)°

MODULATION FACTOR:

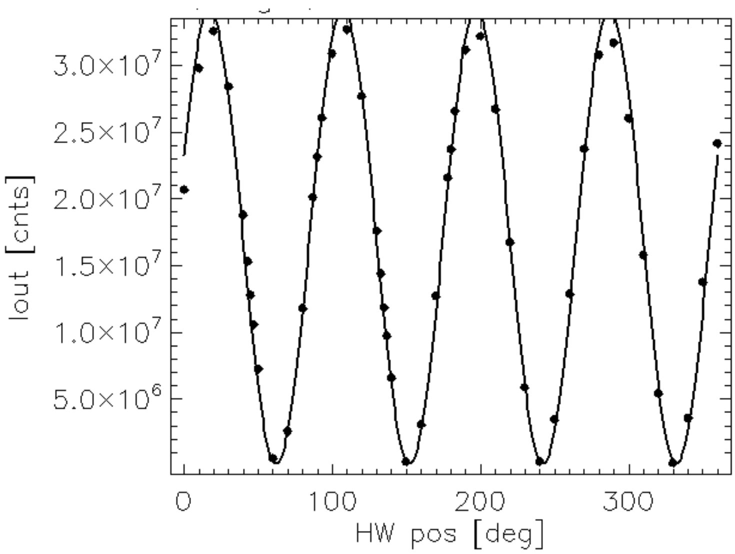
μ = 0.99

V_{LCVR} = 4500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16938966\pm0.04\times10^7$

b = 176.97 ± 0.3

c = $17129906\pm0.3\times10^6$

PHASE:

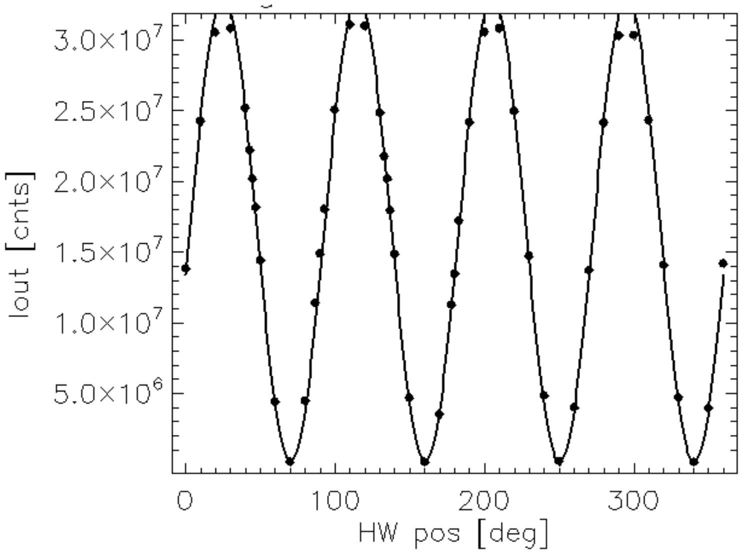
Φ = b - 30° =

(146.97±0.3)°

MODULATION FACTOR:

μ = 0.99

V_{LCVR} = 5000 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s

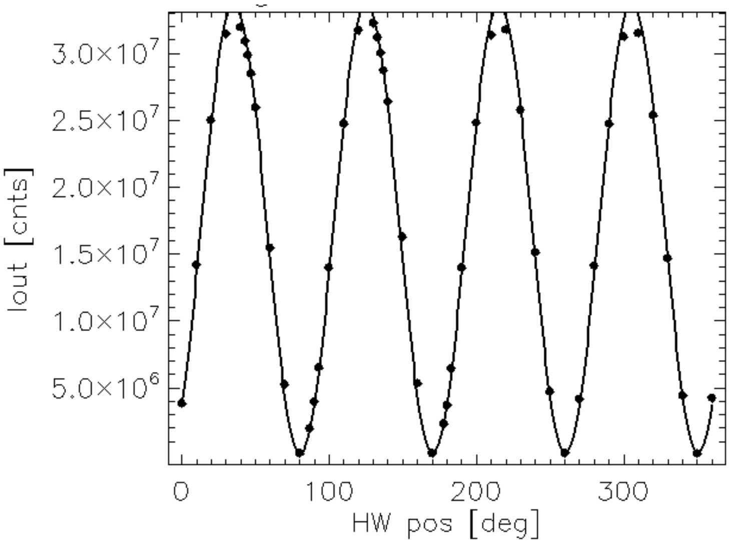


FIT PARAMETERS:
a = -16372841±0.03×10^7
b = 160.83±0.2
c = 16553062±1.6×10^5

PHASE:
Φ = b - 30° =
(130.83±0.2)°

MODULATION FACTOR:
μ = 0.99

V_{LCVR} = 5400 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s



FIT PARAMETERS:
a = -16792234±0.02×10^7
b = 140.43±0.2
c = 16908928±1.3×10^5

PHASE:
Φ = b - 30° =
(110.43±0.2)°

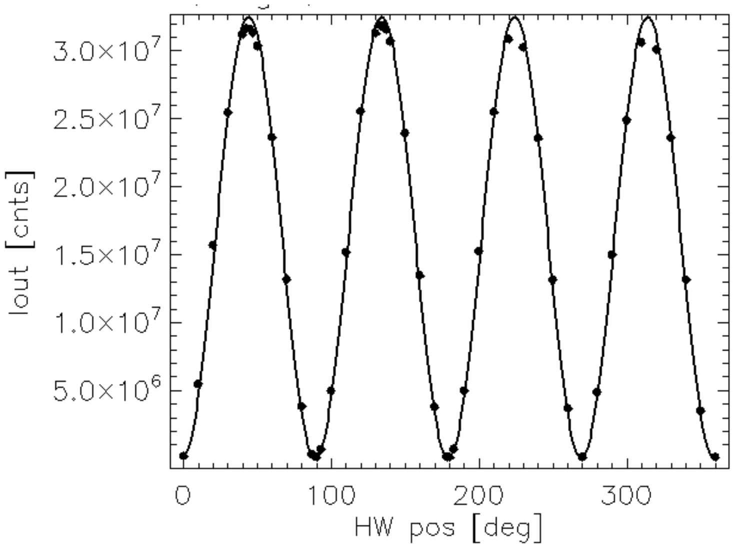
MODULATION FACTOR:
μ = 1.00

V_{LCVR} = 6000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16191920±0.02×10^7

b = 123.05±0.2

c = 16272057±1.1×10^5

PHASE:

Φ = b - 30° = (93.05±0.2)°

MODULATION FACTOR:

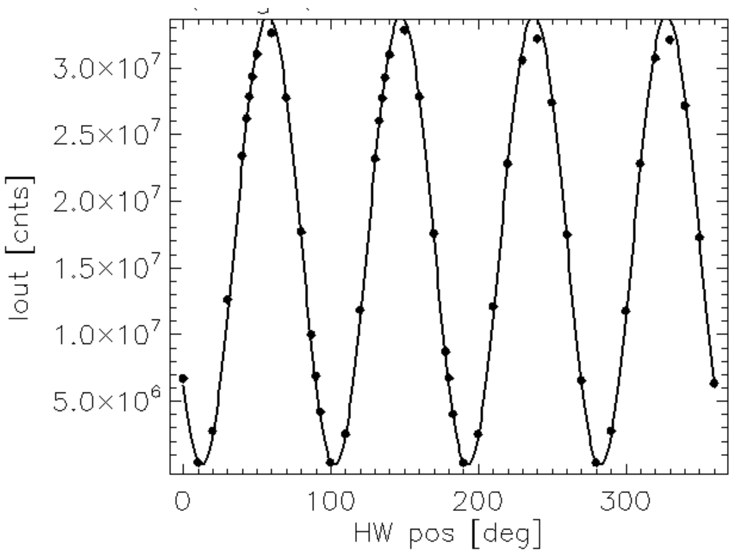
μ = 1.00

V_{LCVR} = 7000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16819690±0.02×10^7

b = 96.36±0.2

c = 17032629±1.2×10^5

PHASE:

Φ = b - 30° = (66.36±0.2)°

MODULATION FACTOR:

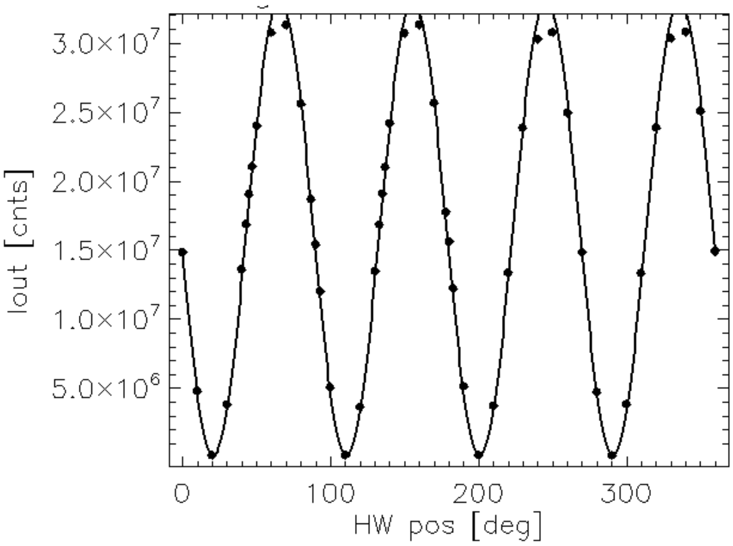
μ = 0.98

V_{LCVR} = 8000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16479350 \pm 0.02\times10^7$

b = 79.80 ± 0.2

c = $16612765 \pm 1.4\times10^5$

PHASE:

Φ = b - 30° = (49.80±0.2)°

MODULATION FACTOR:

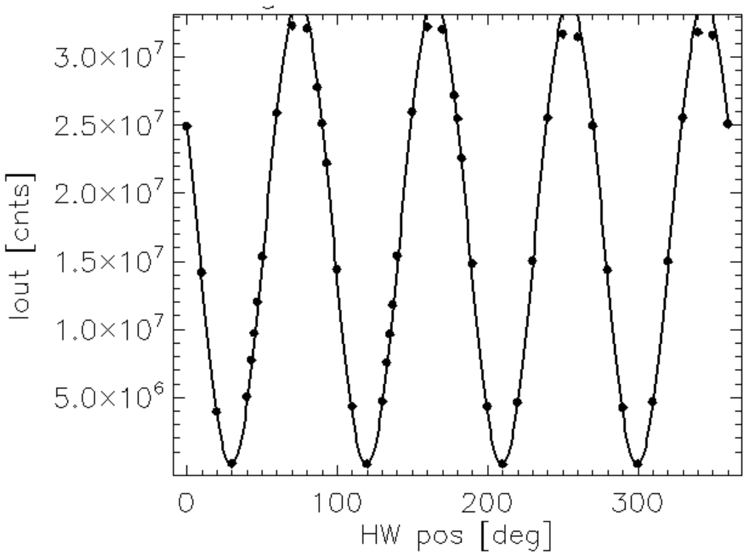
μ = 0.99

V_{LCVR} = 9000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-17008351 \pm 0.02\times10^7$

b = 62.29 ± 0.2

c = $17129981 \pm 1.3\times10^5$

PHASE:

Φ = b - 30° = (32.29±0.2)°

MODULATION FACTOR:

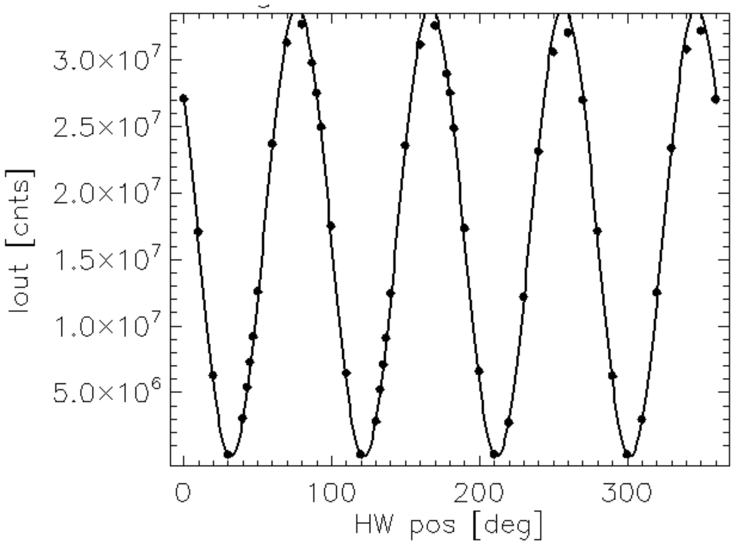
μ = 0.99

V_{LCVR} = 10000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = - 16854486±0.02×10^7

b = 57.50±0.2

c = 17079172±1.3×10^5

PHASE:

Φ = b - 30° =

(27.50±0.2)°

MODULATION FACTOR:

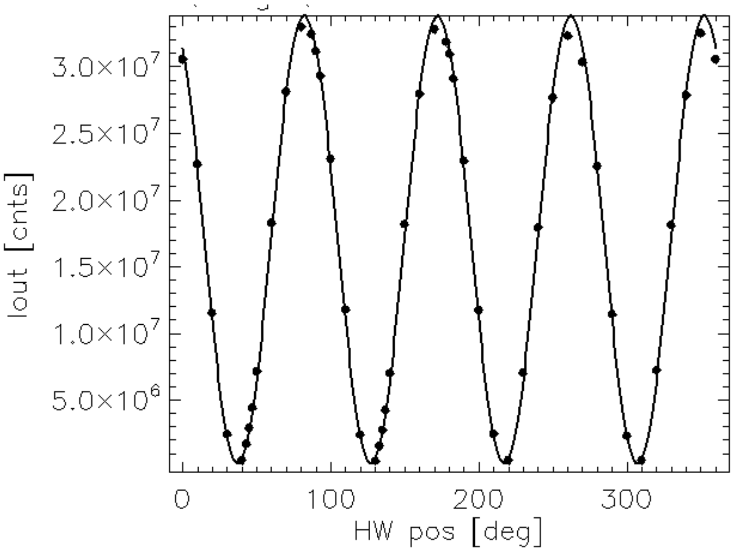
μ = 0.98

V_{LCVR} = 11000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = - 16793024±0.02×10^7

b = 47.04±0.2

c = 17037564±1.1×10^5

PHASE:

Φ = b - 30° =

(17.04±0.2)°

MODULATION FACTOR:

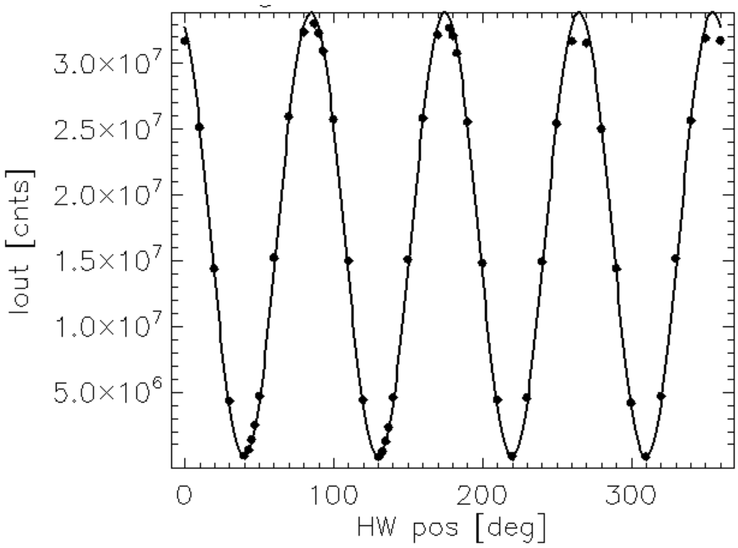
μ = 0.97

V_{LCVR} = 12000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = - 16891742±0.02×10^7

b = 41.87±0.2

c = 16987663±1.1×10^5

PHASE:

Φ = b - 30° = (11.87±0.2)°

MODULATION FACTOR:

μ = 1.00

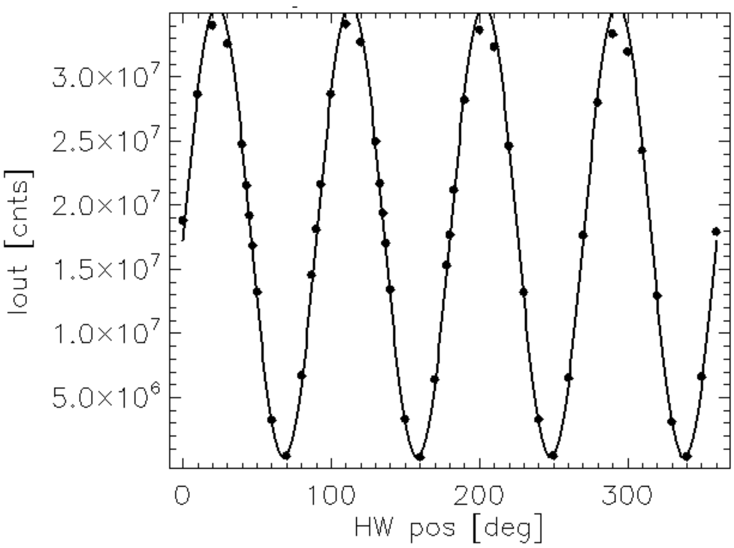
T_{LCVR} = 30°C

V_{LCVR} = 0 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17678369±0.03×10^7

b = 255.18±0.2

c = 17991499±0.2×10^6

PHASE:

Φ = b + 60° = (315.18±0.2)°

MODULATION FACTOR:

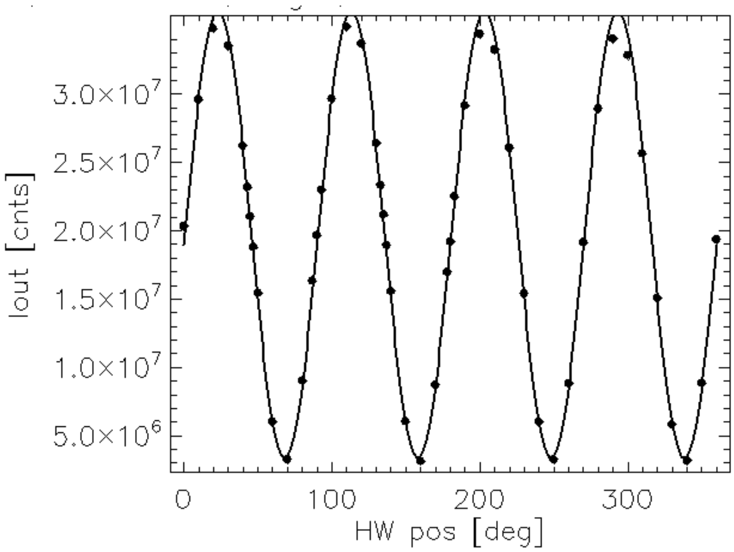
μ = 0.98

V_{LCVR} = 1000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16352682 \pm 0.02\times10^{7}$

b = 254.95 ± 0.2

c = $19727283 \pm 0.1\times10^{6}$

PHASE:

$\Phi = b + 60° = (314.95 \pm 0.2)°$

MODULATION FACTOR:

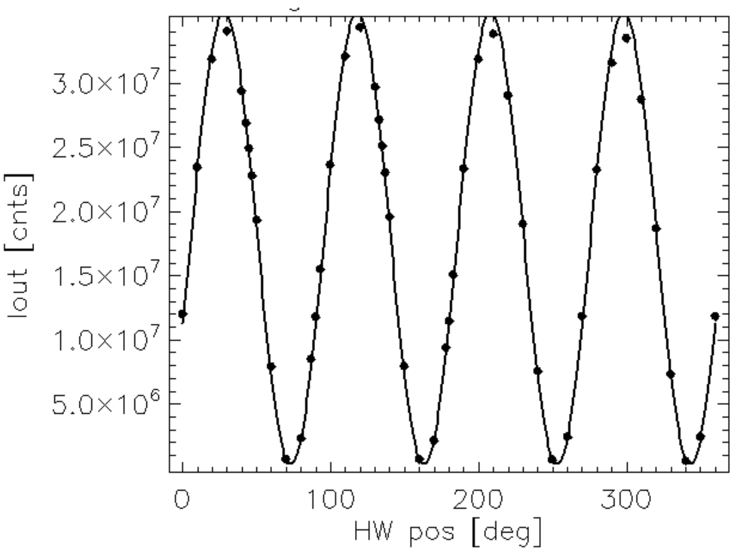
$\mu = 0.84$

V_{LCVR} = 2000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-17556897 \pm 0.02\times10^{7}$

b = 245.27 ± 0.2

c = $17894747 \pm 1.5\times10^{5}$

PHASE:

$\Phi = b + 60° = (305.27 \pm 0.2)°$

MODULATION FACTOR:

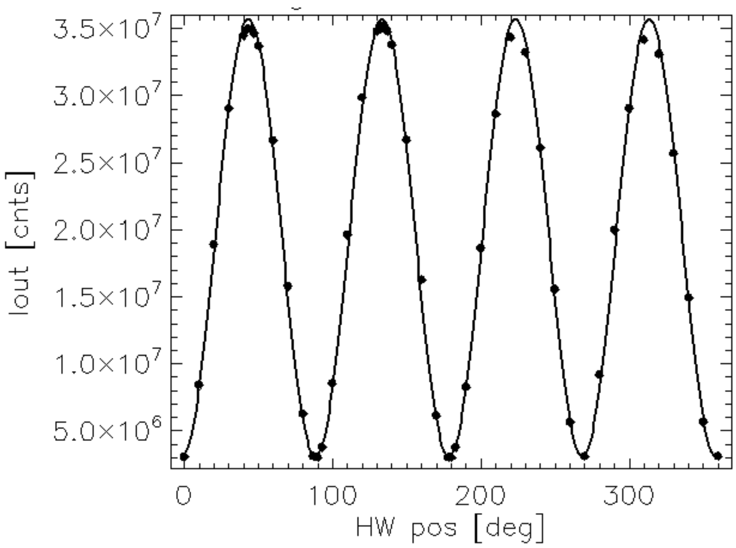
$\mu = 0.97$

V_{LCVR} = 2500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16275603 \pm 0.02\times10^7$

b = 214.57 ± 0.2

c = $19399319. \pm 0.1\times10^6$

PHASE:

$\Phi = b + 60° = (274.57 \pm 0.2)°$

MODULATION FACTOR:

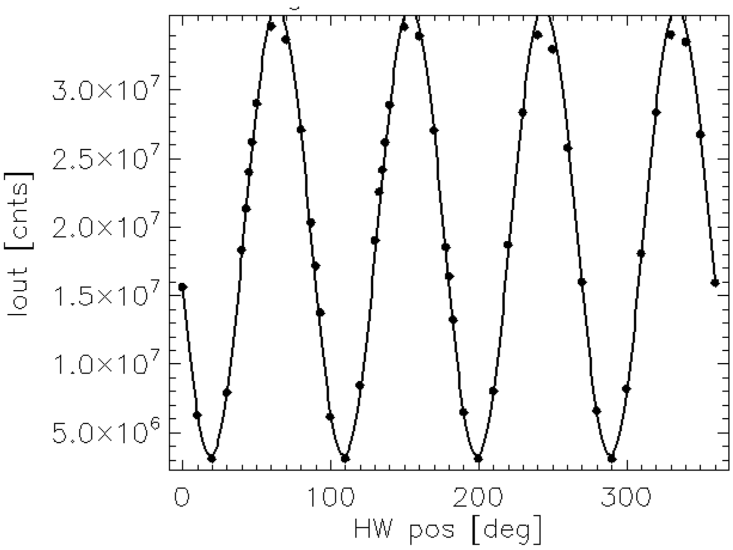
$\mu = 0.84$

V_{LCVR} = 3000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16335506 \pm 0.03\times10^7$

b = 173.32 ± 0.2

c = $19659573 \pm 0.2\times10^6$

PHASE:

$\Phi = b + 60° = (233.32 \pm 0.2)°$

MODULATION FACTOR:

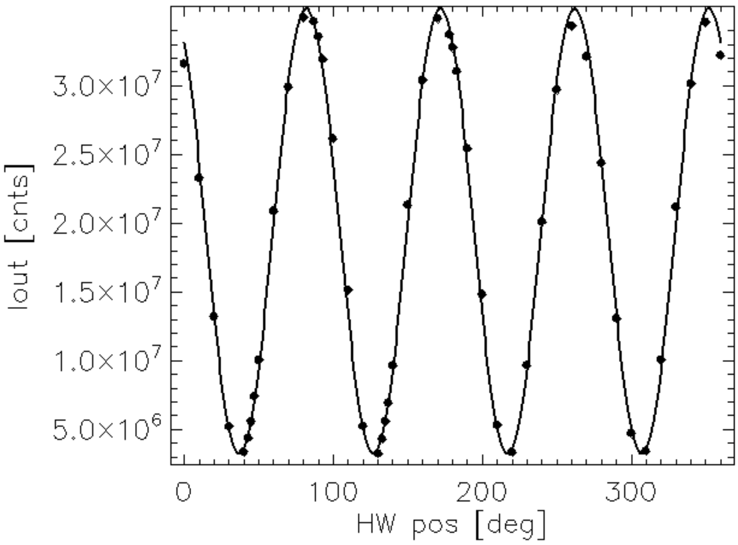
$\mu = 0.84$

V_{LCVR} = 3500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16221721±0.03×10^7

b = 137.35±0.3

c = 19418001±0.2×10^6

PHASE:

Φ = b + 60° =

(197.35±0.3)°

MODULATION FACTOR:

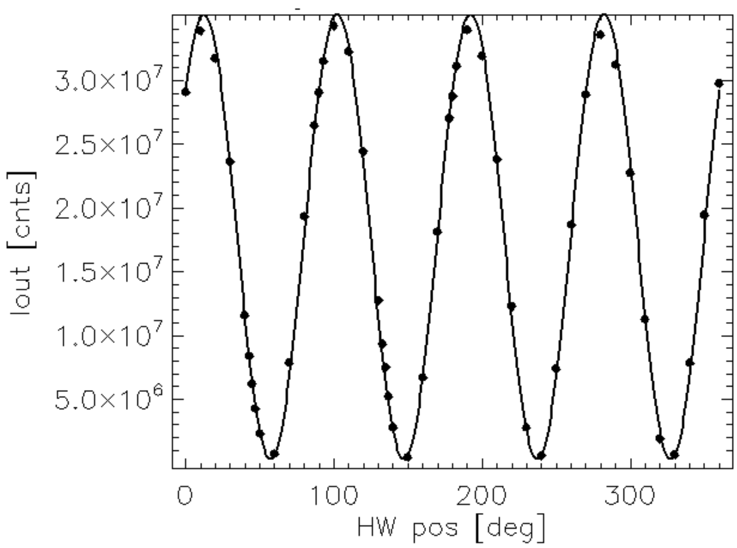
μ = 0.83

V_{LCVR} = 4000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17430886±0.03×10^7

b = 96.88±0.2

c = 17753995±1.6×10^5

PHASE:

Φ = b + 60° =

(156.88±0.2) °

MODULATION FACTOR:

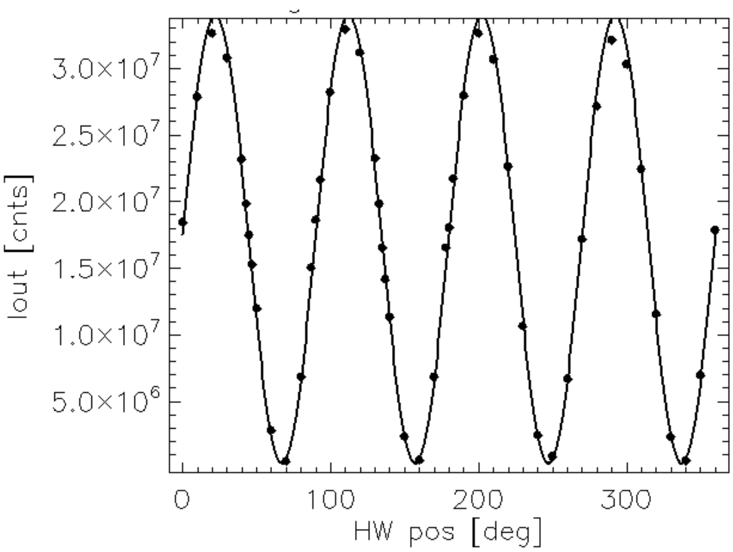
μ = 0.98

V_{LCVR} = 4500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16840638±0.03×10^7

b = 76.98±0.3

c = 17172366±1.7×10^5

PHASE:

Φ = b + 60° =

(136.98±0.3) °

MODULATION FACTOR:

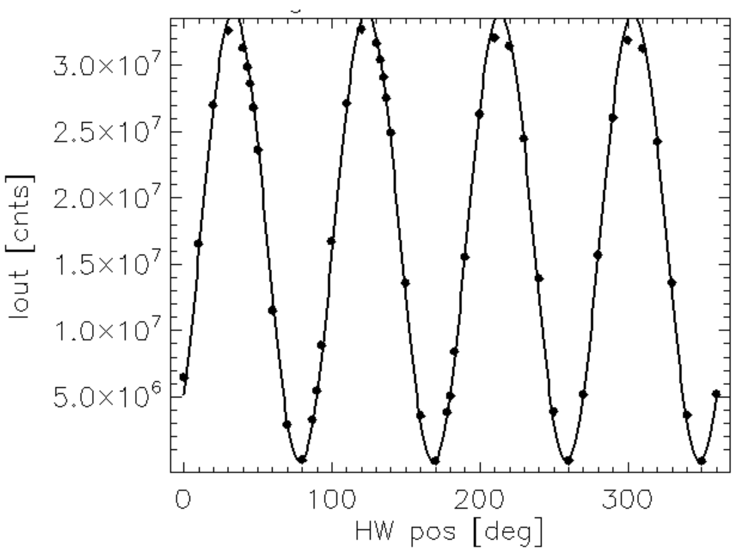
μ = 0.97

V_{LCVR} = 5000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16890413±0.02×10^7

b = 54.05±0.2

c = 17010615±1.5×10^5

PHASE:

Φ = b + 60° =

(114.05±0.2)°

MODULATION FACTOR:

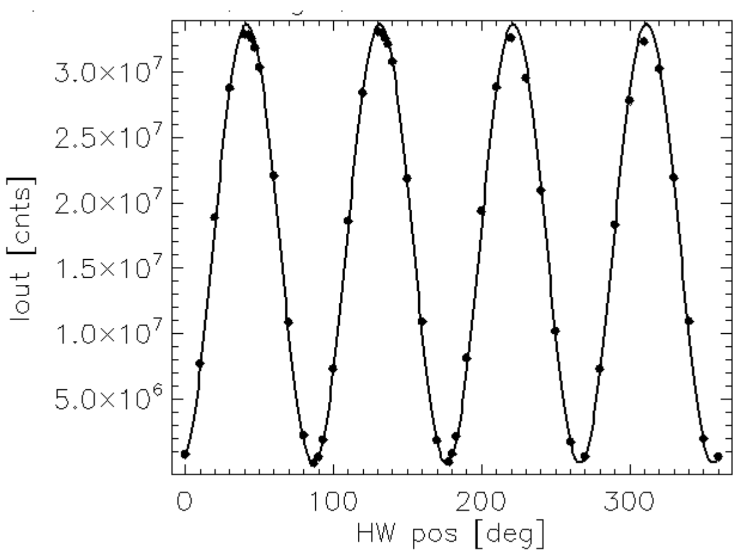
μ = 0.99

V_{LCVR} = 5400 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = −16757424±0.02×10^7^

b = 38.51±0.2

c = 16870150±1.2×10^5^

PHASE:

Φ = b + 60° =

(98.51±0.2) °

MODULATION FACTOR:

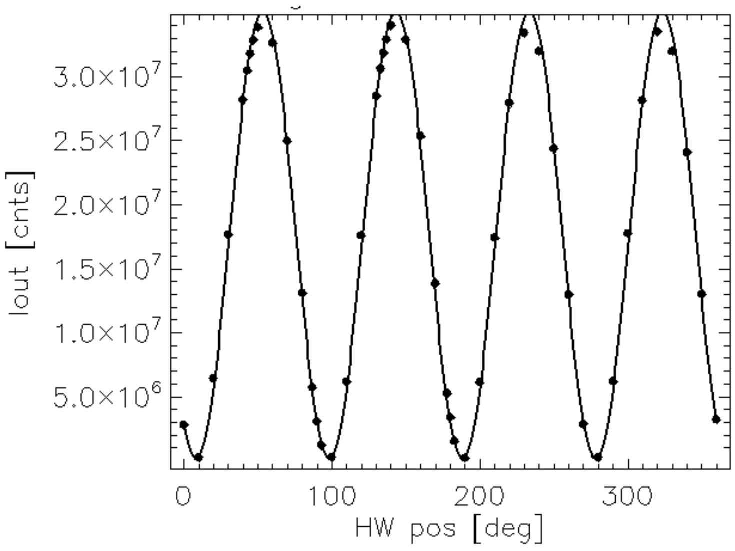
μ = 1.00

V_{LCVR} = 6000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = −17472594±0.02×10^7^

b = 14.75±0.2

c = 17638667±1.3×10^5^

PHASE:

Φ = b + 60° =

(74.75±0.2)°

MODULATION FACTOR:

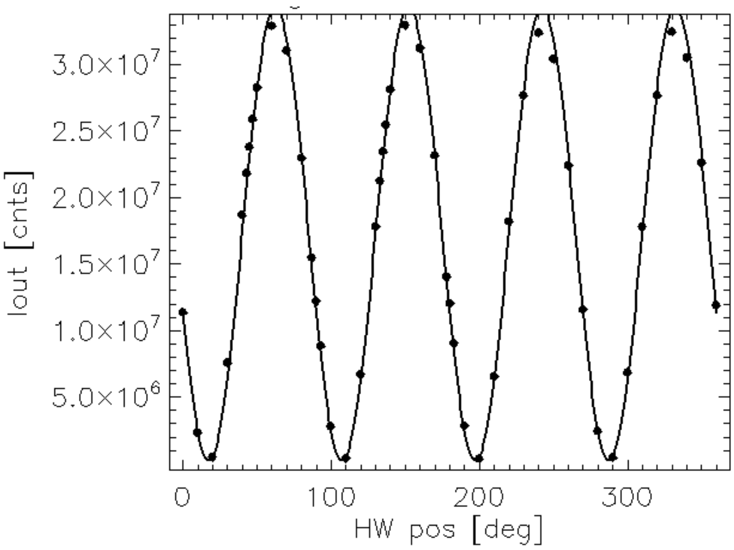
μ = 0.99

V_{LCVR} = 7000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17017977±0.03×10^7

b = -3.54±0.2

c = 17241989±1.6×10^5

PHASE:

Φ = b + 60° =

(56.46±0.2)°

MODULATION FACTOR:

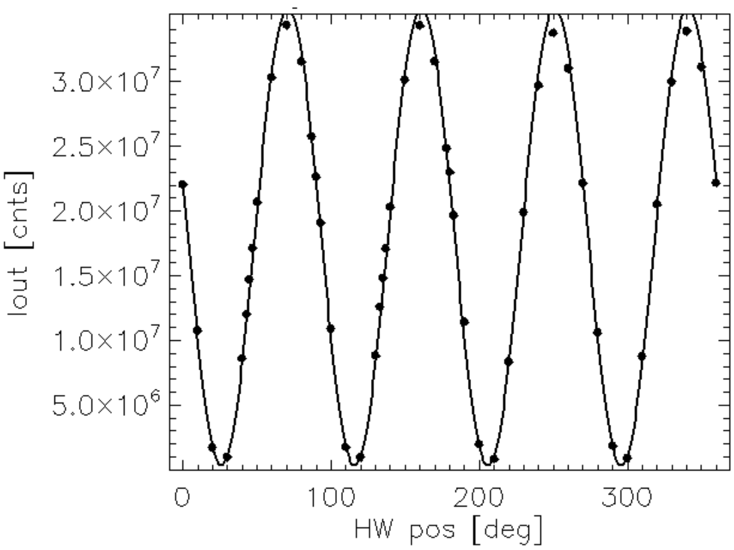
μ = 0.98

V_{LCVR} = 8000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17665939±0.03×10^7

b = -20.47±0.2

c = 18003721±0.2×10^6

PHASE:

Φ = b + 60° =

(39.53±0.2)°

MODULATION FACTOR:

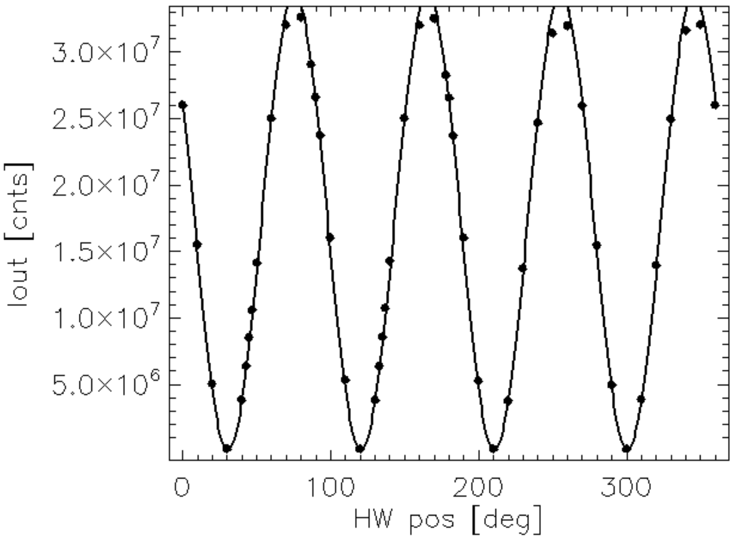
μ = 0.95

V_{LCVR} = 9000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17042918±0.02×10^7

b = -29.90±0.2

c = 17185062±1.3×10^5

PHASE:

Φ = b + 60° = (30.01±0.2)°

MODULATION FACTOR:

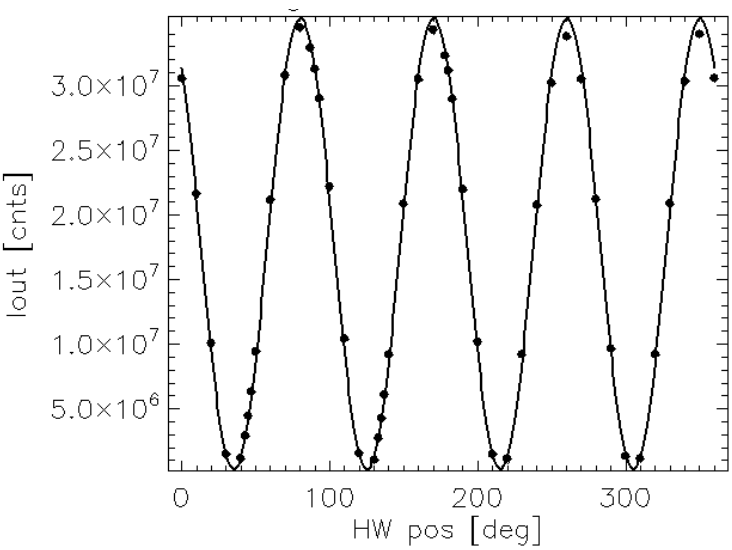
μ = 0.99

V_{LCVR} = 10000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17427744±0.02×10^7

b = -39.26±0.2

c = 17762147±1.2×10^5

PHASE:

Φ = b + 60° = (20.74±0.2)°

MODULATION FACTOR:

μ = 0.94

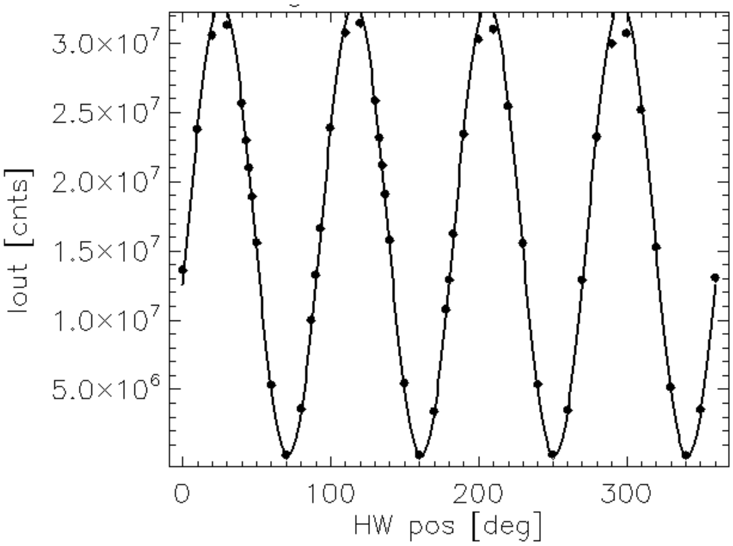
T_{LCVR} = 34°C

V_{LCVR} = 0 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16372693 \pm 0.02\times10^{7}$

b = 339.25 ± 0.2

c = $16610146 \pm 1.4\times10^{5}$

PHASE:

Φ = b - 30° =

(309.25±0.2) °

MODULATION FACTOR:

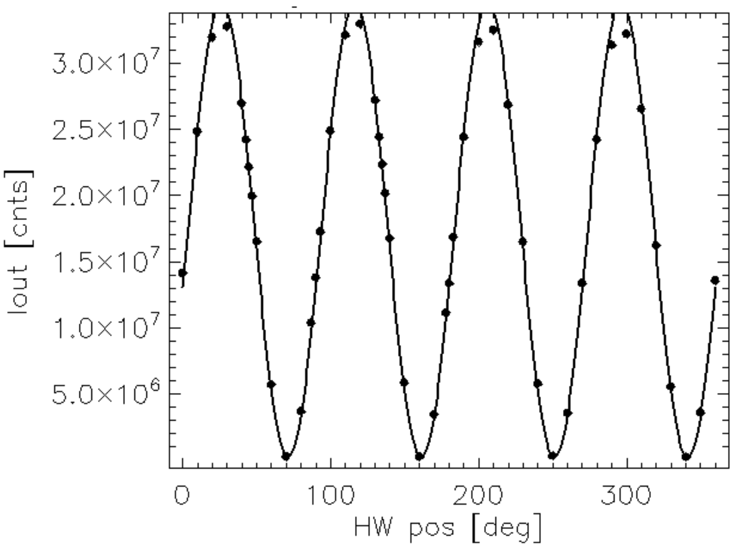
μ = 0.99

V_{LCVR} = 1000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-17132518 \pm 0.02\times10^{7}$

b = 338.98 ± 0.2

c = $17396566 \pm 1.3\times10^{5}$

PHASE:

Φ = b - 30° =

(308.98±0.2) °

MODULATION FACTOR:

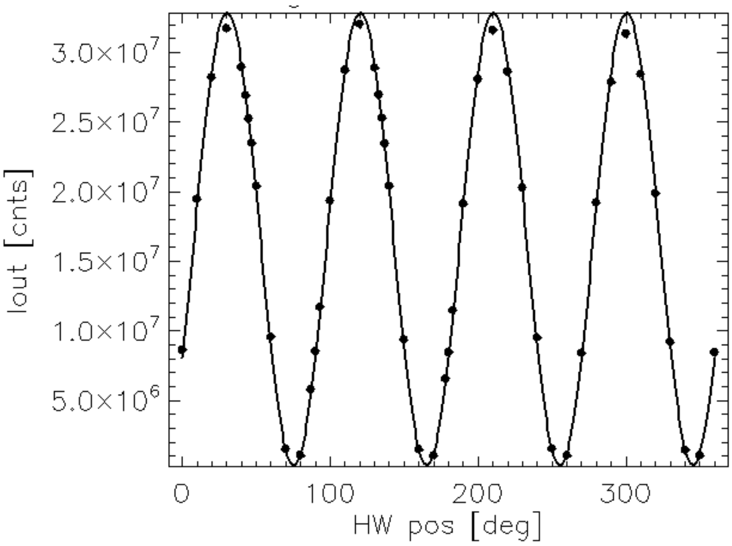
μ = 0.99

V_{LCVR} = 2000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16219196\pm0.02\times10^7$

b = 330.52 ± 0.2

c = $16599040\pm1.2\times10^5$

PHASE:

Φ = b - 30° =
(300.52±0.2) °

MODULATION FACTOR:

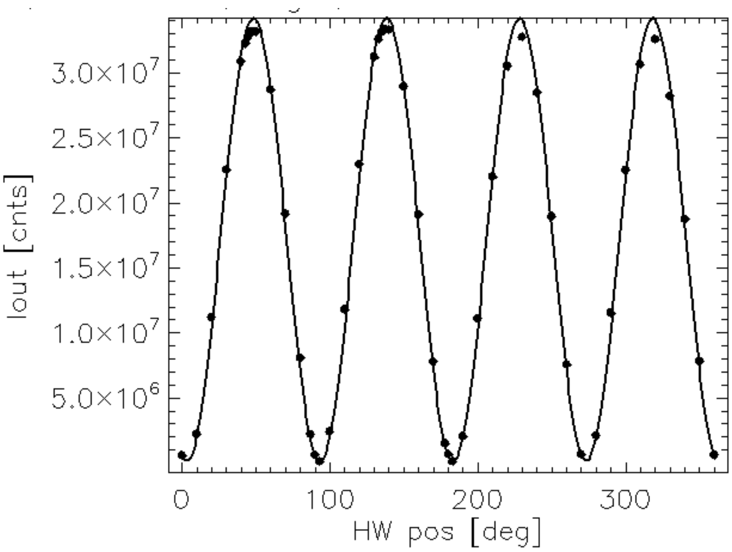
μ = 0.94

V_{LCVR} = 2500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-17000966\pm0.02\times10^7$

b = 294.64 ± 0.2

c = $17171564\pm1.4\times10^5$

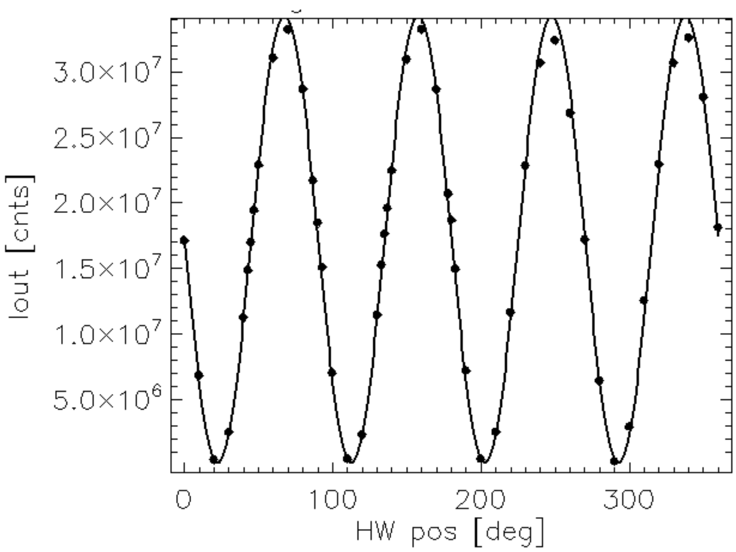
PHASE:

Φ = b - 30° =
(264.64±0.2) °

MODULATION FACTOR:

μ = 0.99379123

V_{LCVR} = 3000 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s

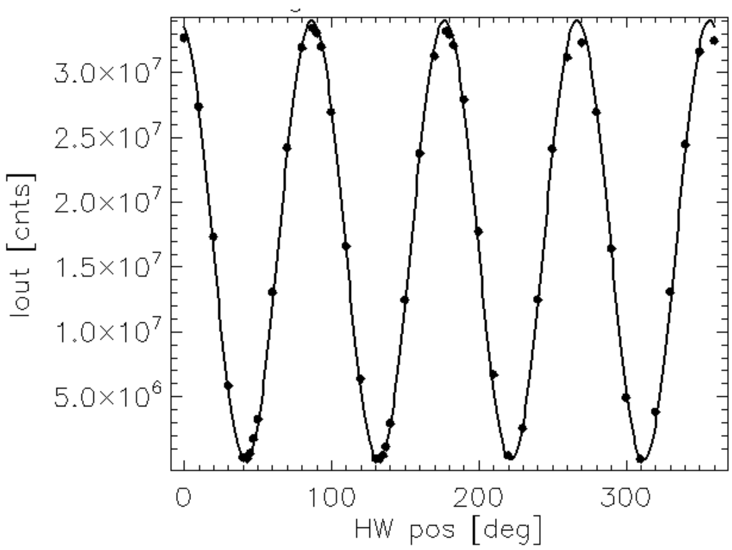


FIT PARAMETERS:
a = −17049832±0.03×10^7
b = 255.99±0.2
c = 17246173±1.7×10^5

PHASE:
Φ = b − 30° = (225.99±0.2) °

MODULATION FACTOR:
μ = 0.98

V_{LCVR} = 3500 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s



FIT PARAMETERS:
a = −16924552±0.02×10^7
b = 217.92±0.3
c = 17072796±1.3×10^5

PHASE:
Φ = b − 30° = (187.92±0.3) °

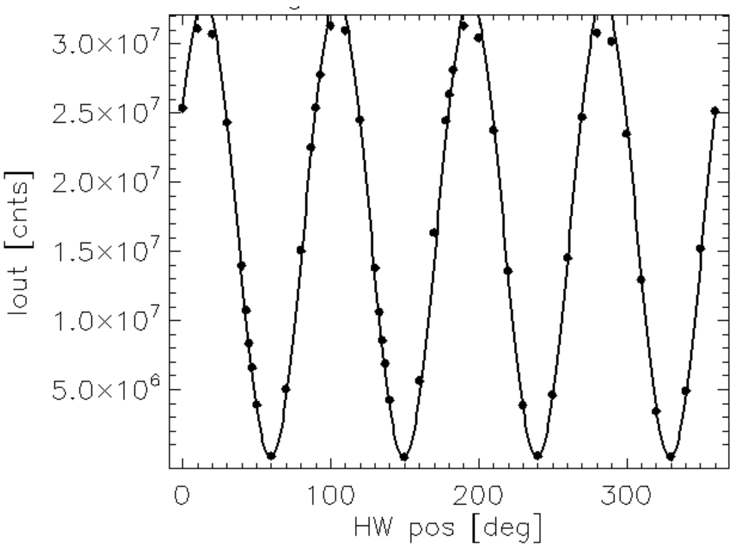
MODULATION FACTOR:
μ = 0.99

V_{LCVR} = 4000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16450024±0.03×10^7

b = 182.39±0.2

c = 16633946±1.6×10^5

PHASE:

Φ = b - 30° =

(152.39±0.2) °

MODULATION FACTOR:

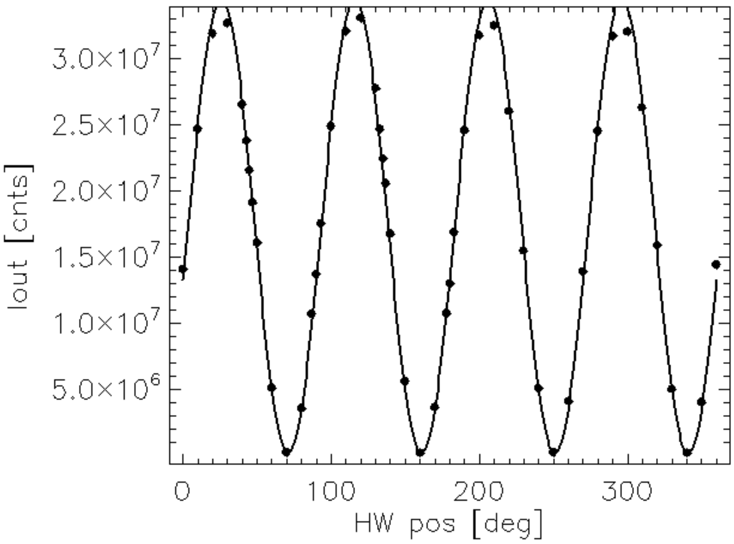
μ = 0.99

V_{LCVR} = 4500 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17134384±0.03×10^7

b = 159.52±0.3

c = 17312404±1.7×10^5

PHASE:

Φ = b - 30° =

(129.52±0.3) °

MODULATION FACTOR:

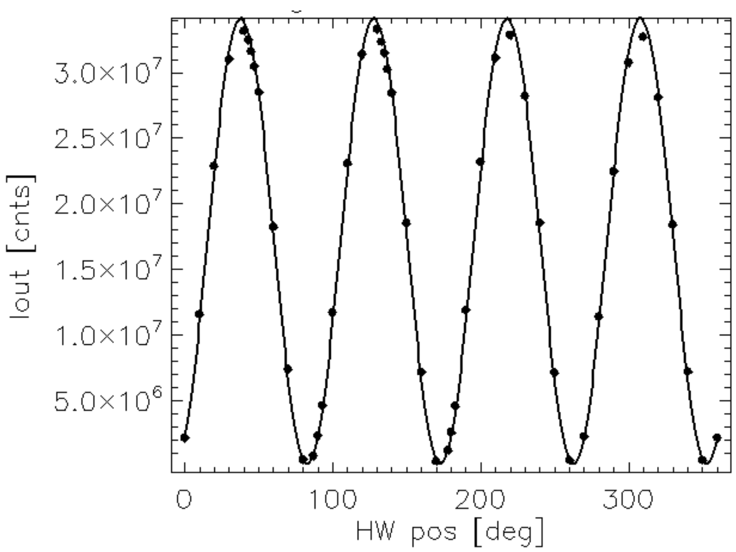
μ = 0.99

V_{LCVR} = 5000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-16954660 \pm 0.02 \times 10^7$

b = 135.69 ± 0.2

c = $17179351 \pm 1.2 \times 10^5$

PHASE:

$\Phi = b - 30° =$ (105.69 ± 0.2) °

MODULATION FACTOR:

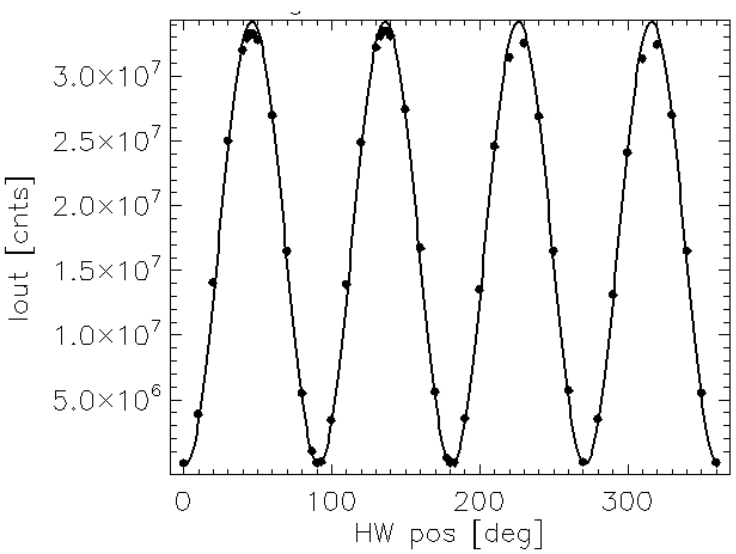
$\mu = 0.98$

V_{LCVR} = 5400 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = $-17060286 \pm 0.02 \times 10^7$

b = 118.87 ± 0.2

c = $17140209 \pm 1.3 \times 10^5$

PHASE:

$\Phi = b - 30° =$ (88.87 ± 0.2) °

MODULATION FACTOR:

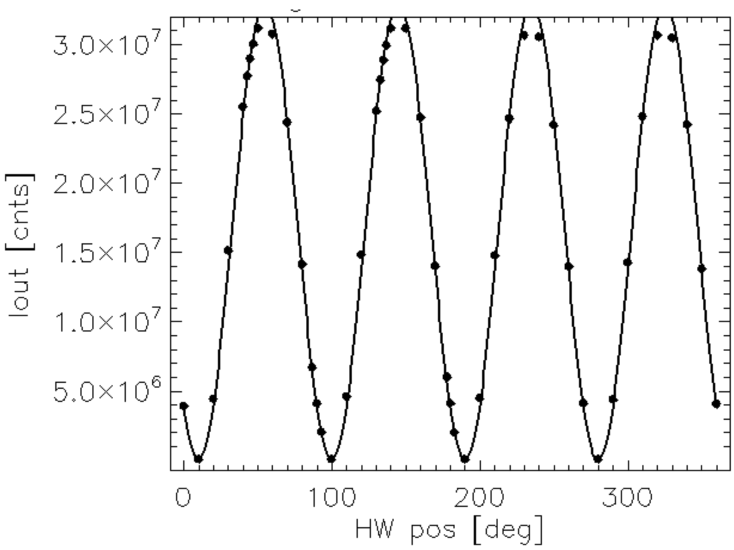
$\mu = 1.00$

V_{LCVR} = 6000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -16390185±0.02×10^7

b = 101.66±0.2

c = 16476745±1.2×10^5

PHASE:

Φ = b - 30° =

(71.66±0.2) °

MODULATION FACTOR:

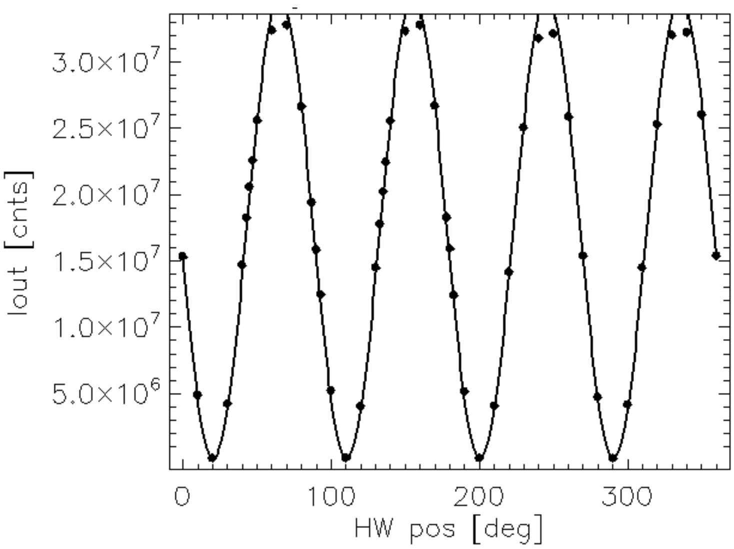
μ = 1.00

V_{LCVR} = 7000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = -17315987±0.03×10^7

b = 80.29±0.2

c = 17438257±1.6×10^5

PHASE:

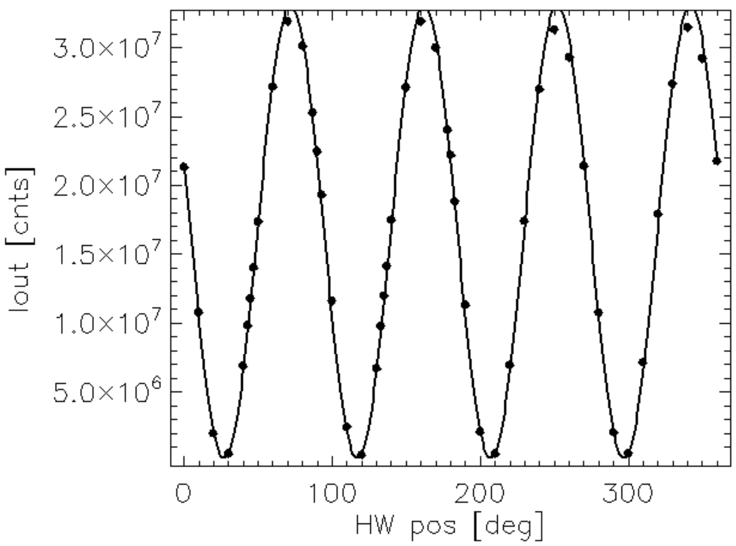
Φ = b - 30° =

(50.29±0.2) °

MODULATION FACTOR:

μ = 0.99

V_{LCVR} = 8000 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s

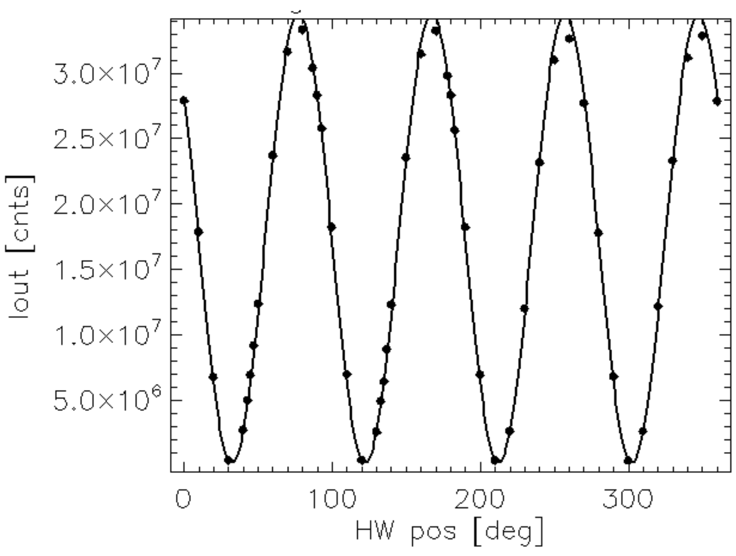


FIT PARAMETERS:
a = -16397387±0.02×10^7
b = 67.31±0.2
c = 16602955±1.4×10^5

PHASE:
Φ = b - 30° =
(37.31±0.2) °

MODULATION FACTOR:
μ = 0.97

V_{LCVR} = 9000 mV
λ_o = 620 nm
$\Delta\lambda$ = 10 nm
Exposition time = 10 s



FIT PARAMETERS:
a = -17113148±0.02×10^7
b = 56.66±0.2
c = 17355460±1.3×10^5

PHASE:
Φ = b - 30° =
(26.66±0.2) °

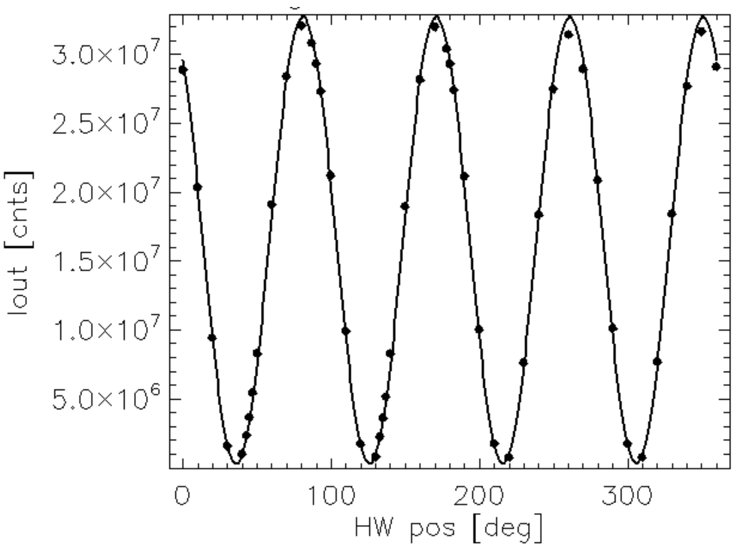
MODULATION FACTOR:
μ = 0.98

V_{LCVR} = 10000 mV

λ_o = 620 nm

$\Delta\lambda$ = 10 nm

Exposition time = 10 s



FIT PARAMETERS:

a = −16206078±0.02×10^7

b = 49.47±0.2

c = 16502505±1.1×10^5

PHASE:

Φ = b - 30° =

(19.47±0.2) °

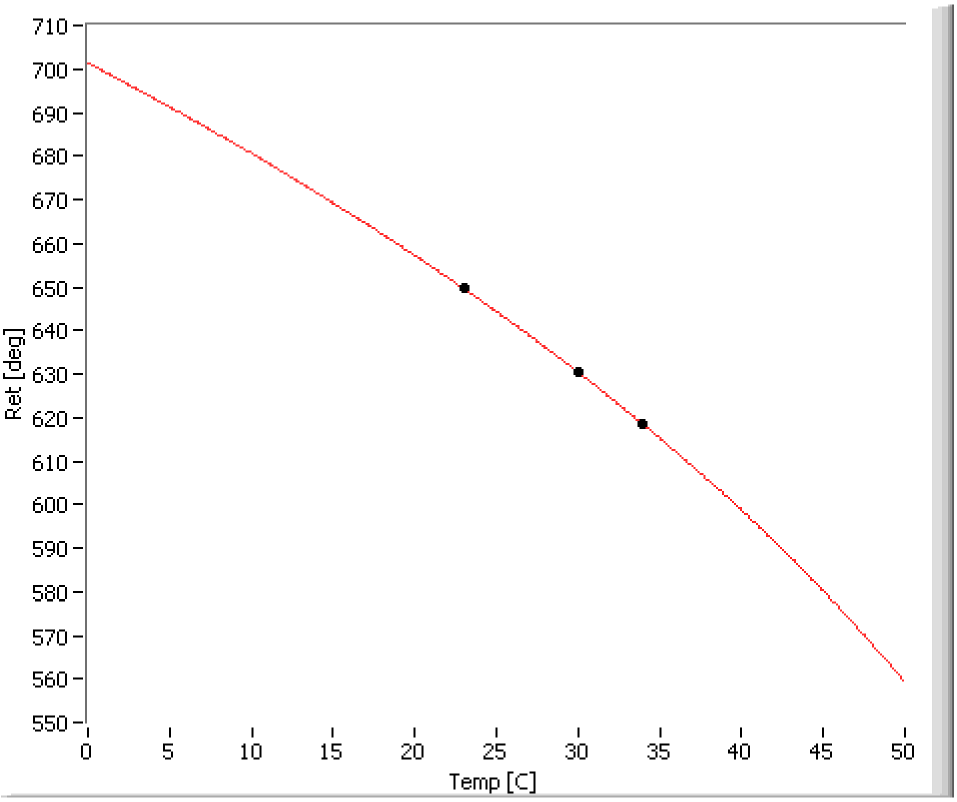
MODULATION FACTOR:

μ = 0.96

Appendix B- Rotation vs. Temperature Data Analysis

Fit for V= 0V

Voltage applied to LCVR: 0mV lambda = white light Date: May 2008		
Temperature [°C]	**Retardance [deg]**	**Err Ret. [deg]**
23	649.6	0.4
30	630.4	0.4
34	618.4	0.4



Fit return this values:

$$\begin{cases} \delta_0 = 701 \pm 1.4 \deg \\ T_C = 76.9 \pm 0.2°C \\ \beta = 0.216 \pm 0.004 \end{cases}$$

In graphics, the black points are the experimental data and the red line is the fit curve.

The values of δ_0 are acceptable and the values of T_C and β are compatible with the bibliography [4]. Since there aren't further details from the supplier, the results seem to be acceptable.

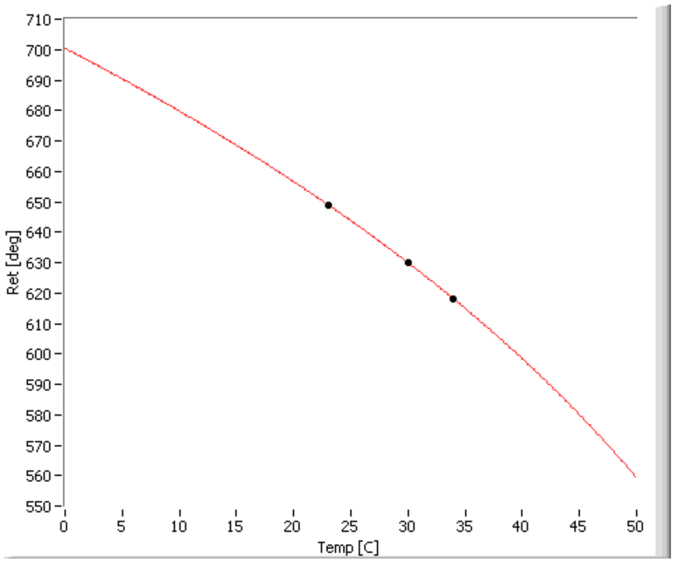
For the other fits we use only δ_0 e β , while T_C remain constant at 76.9° C. Data are the follows (tab. B1):

T [°C]	Rot[deg]	Ret[deg]	Err Ret [deg]
1000 mV			
23	324.5	649	0.4
30	315	630	0.4
34	309	618	0.4
2000 mV			
23	319.5	639	0.4
30	305.3	610.6	0.4
34	300.5	601	0.4
2500 mV			
23	292.6	585.2	0.4
30	274.6	549.2	0.4
34	264.6	529.2	0.4
3000 mV			
23	255.6	511.2	0.4
30	233.3	466.6	0.4
34	226	452	0.4
3500 mV			
23	218.3	436.6	0.4
30	197.3	394.6	0.4
34	187.9	375.8	0.4
4000 mV			
23	184.9	369.8	0.4
30	156.9	313.8	0.4
34	152.4	304.8	0.4
4500 mV			
23	147	294	0.6
30	137	274	0.6
34	129.5	259	0.6
5000 mV			
23	130.8	261.6	0.4
30	114.1	228.2	0.4
34	105.7	211.4	0.4
5400 mV			
23	110.4	220.8	0.4
30	98.5	197	0.4
34	88.9	177.8	0.4
6000 mV			
23	93.1	186.2	0.4
30	74.7	149.4	0.4
34	71.7	143.4	0.4

7000 mV			
23	66.4	132.8	0.4
30	56.5	113	0.4
34	50.3	100.6	0.4
8000 mV			
23	49.8	99.6	0.4
30	39.5	79	0.4
34	37.3	74.6	0.4
9000 mV			
23	32.3	64.6	0.4
30	30.1	60.2	0.4
34	26.7	53.4	0.4
10000 mV			
23	27.5	55	0.4
30	20.7	41.4	0.4
34	19.5	39	0.4

Tab. B1 – Retardance table.

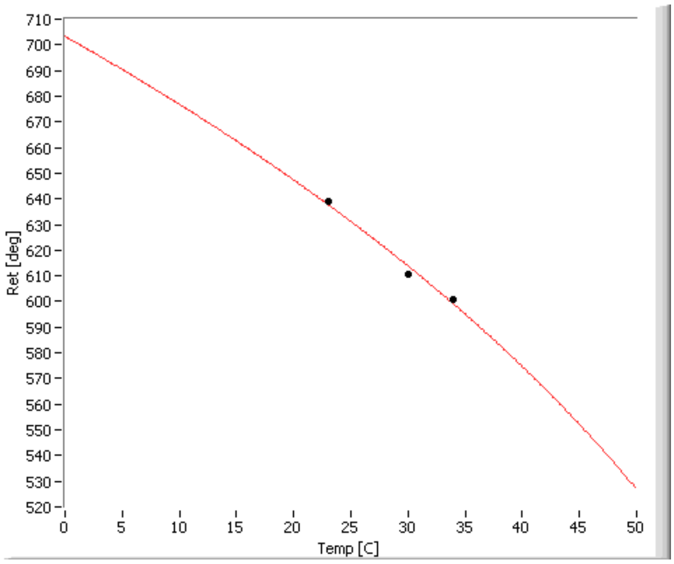
V = 1000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 700 \pm 1.3 \deg \\ \beta = 0.215 \pm 0.004 \end{cases}$$

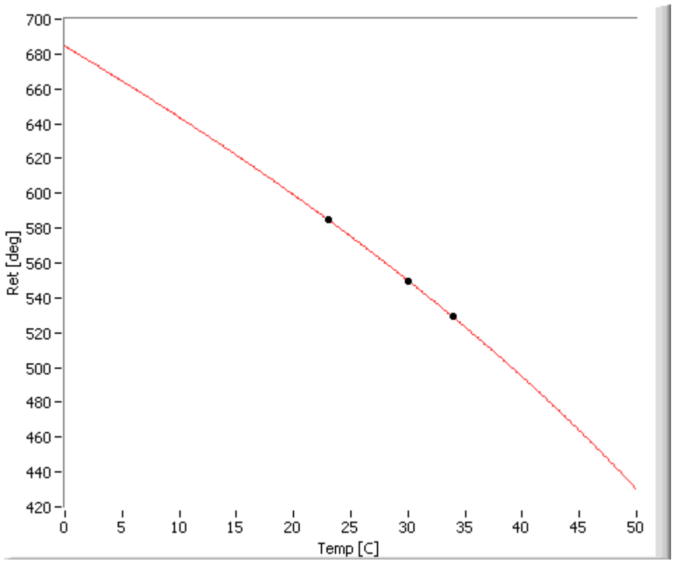
V = 2000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 703 \pm 1.3 \deg \\ \beta = 0.275 \pm 0.004 \end{cases}$$

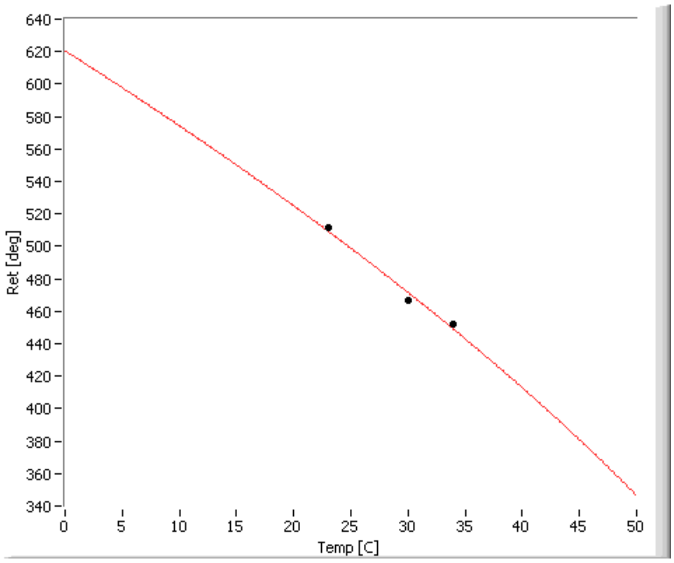
V = 2500 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 684 \pm 1.4 \deg \\ \beta = 0.443 \pm 0.004 \end{cases}$$

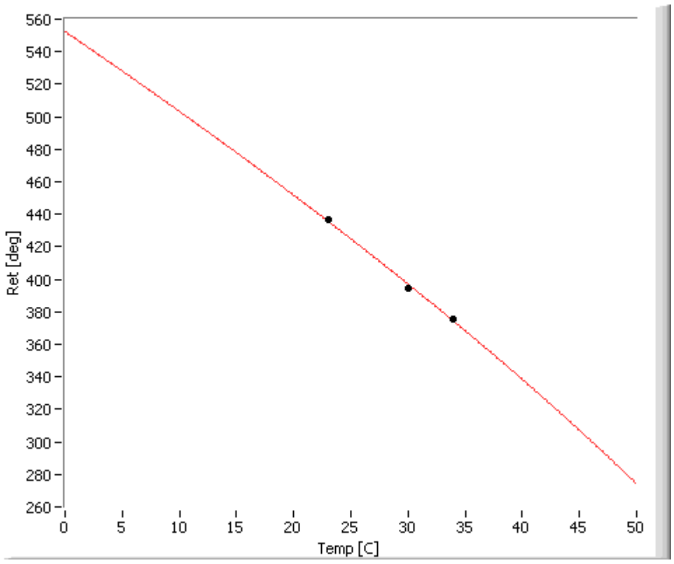
V = 3000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 620 \pm 1.5 \deg \\ \beta = 0.555 \pm 0.005 \end{cases}$$

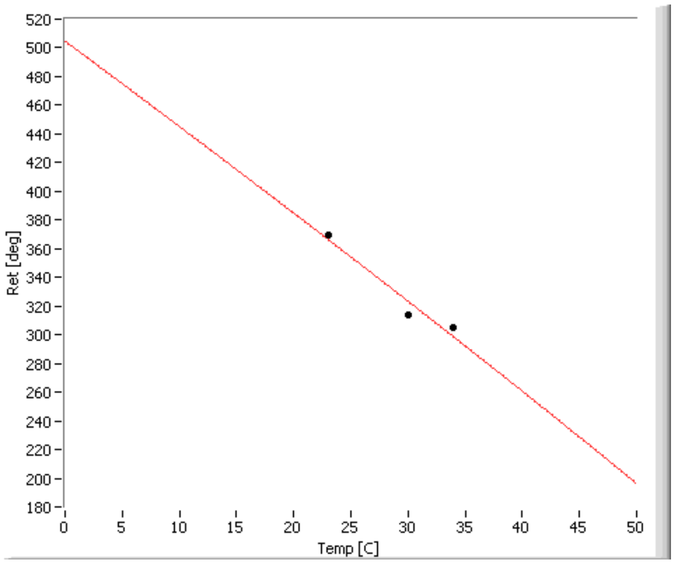
V = 3500 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 552 \pm 1.6 \deg \\ \beta = 0.667 \pm 0.006 \end{cases}$$

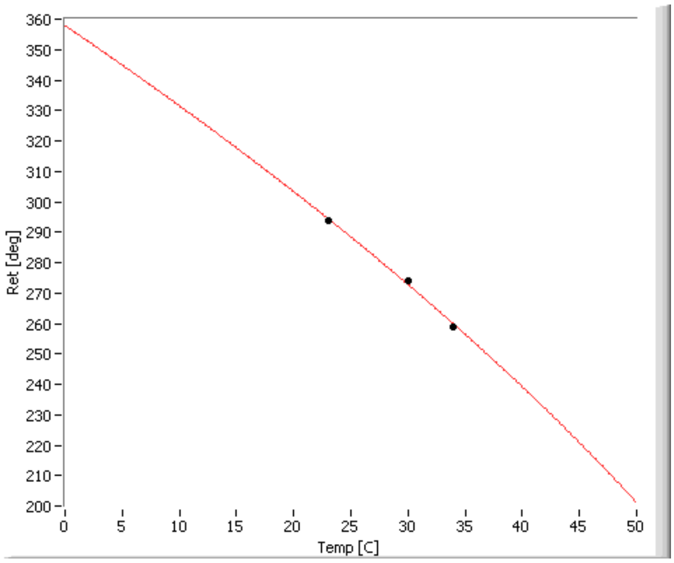
V = 4000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 504 \pm 1.7\,\text{deg} \\ \beta = 0.900 \pm 0.007 \end{cases}$$

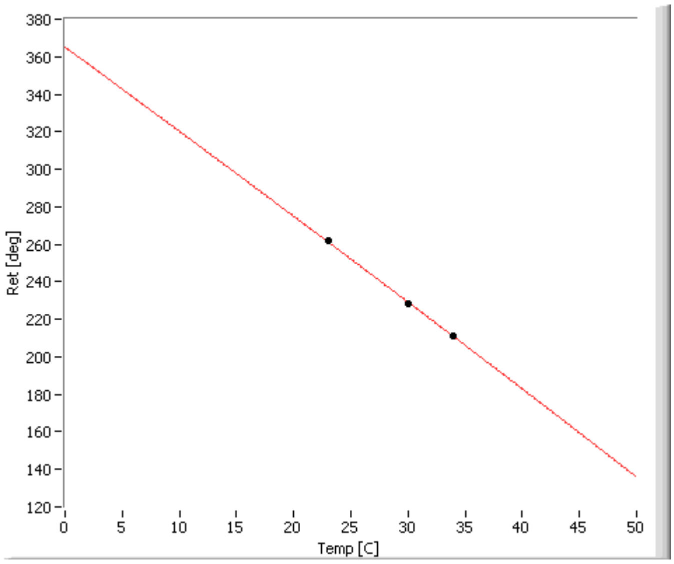
V = 4500 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 358 \pm 2.3\,\text{deg} \\ \beta = 0.55 \pm 0.01 \end{cases}$$

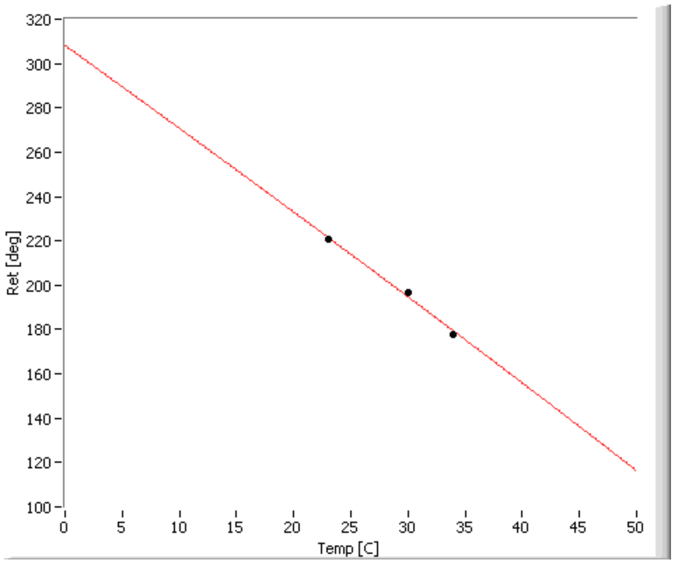
V = 5000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 365 \pm 1.8\,\text{deg} \\ \beta = 0.94 \pm 0.01 \end{cases}$$

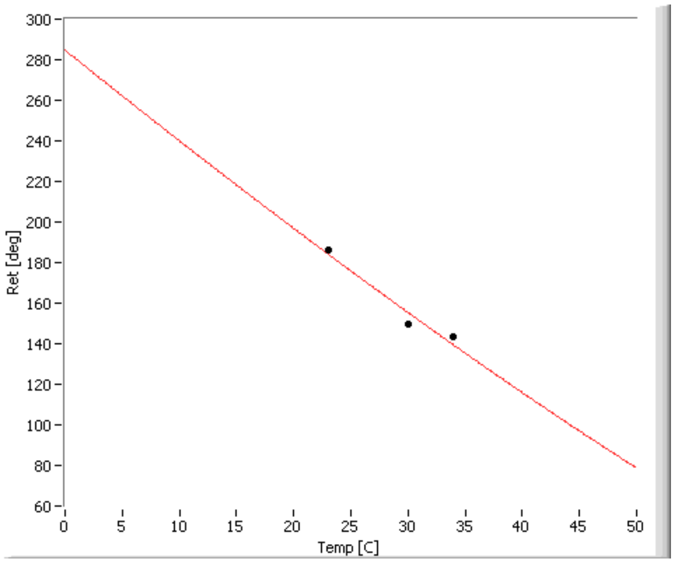
V = 5400 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 308 \pm 1.8\,\text{deg} \\ \beta = 0.93 \pm 0.01 \end{cases}$$

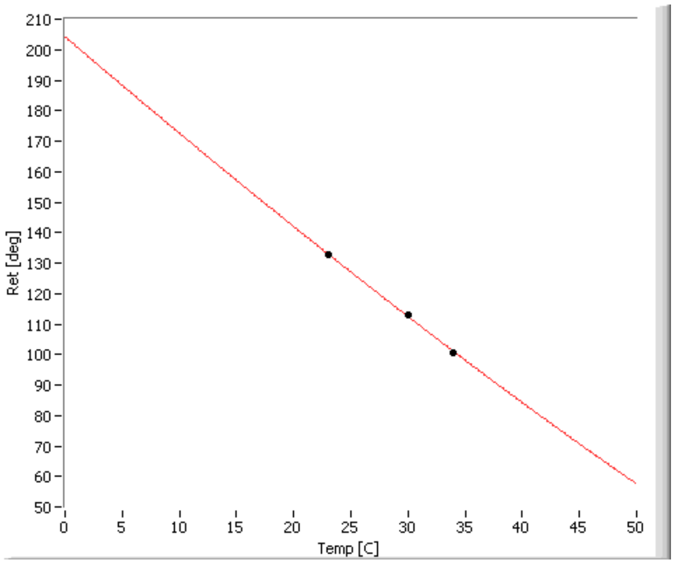
V = 6000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 284 \pm 2\,\text{deg} \\ \beta = 1.22 \pm 0.01 \end{cases}$$

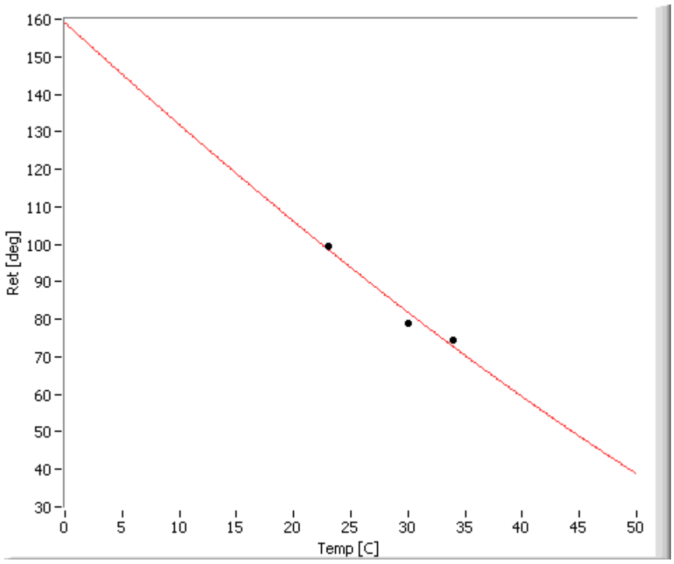
V = 7000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 204 \pm 2\,\text{deg} \\ \beta = 1.21 \pm 0.02 \end{cases}$$

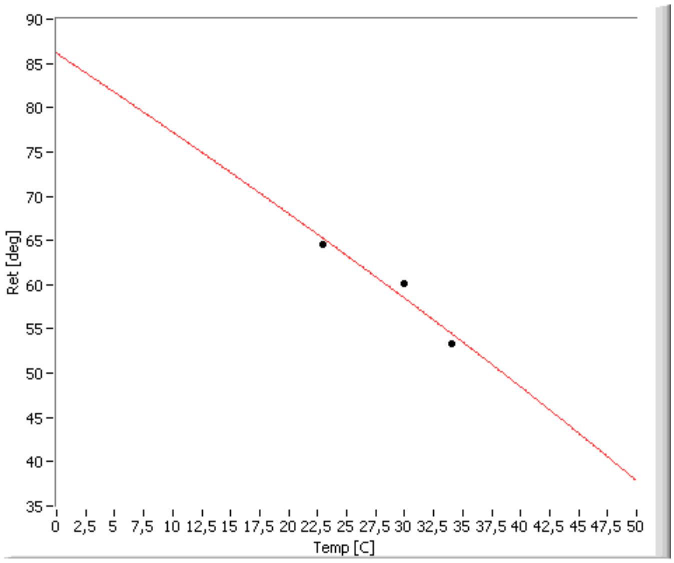
V = 8000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 159 \pm 2.1 \deg \\ \beta = 1.34 \pm 0.03 \end{cases}$$

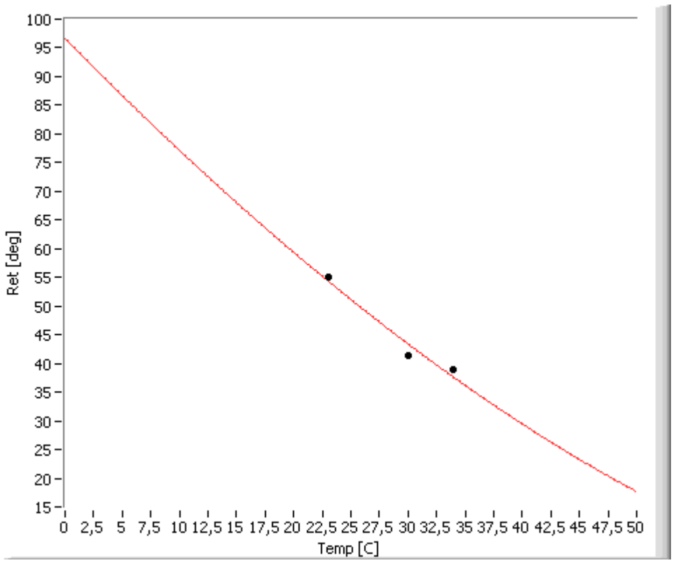
V = 9000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 86 \pm 1.7 \deg \\ \beta = 0.78 \pm 0.04 \end{cases}$$

V = 10000 mV



Fit Parameters:

$$\begin{cases} \delta_0 = 97 \pm 2.4 \deg \\ \beta = 1.62 \pm 0.05 \end{cases}$$

Appendix C – Notes on physics of nematic liquid crystals

The Clausius-Mossotti equation:

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4}{3}\pi\rho\alpha \qquad \text{(C.1)}$$

where ε is the dielectric constant, ρ the molecular density and α the molecular polarizability.

In the optical frequency case:

$$\varepsilon = n^2 \qquad \text{(C.2)}$$

Where n is the refraction index[1]. Replacing C.2 in C.1, we obtain Lorentz-Lorenz equation:

$$\frac{n^2 - 1}{n^2 + 2} = \frac{4}{3}\pi\rho\alpha \qquad \text{(C.3)}$$

For anisotropic liquid crystals, there are two refraction indexes: n_O (ordinary index) and n_S (extraordinary index). They correspond to the molecular polarizability α_O e α_S.

With the Vuks's approach [5], we can write:

$$n_S = \langle n \rangle + \frac{2}{3}\Delta n \qquad \text{(C.4a)}$$

$$n_O = \langle n \rangle - \frac{1}{3}\Delta n \qquad \text{(C.4b)}$$

Where

$$\langle n^2 \rangle = \frac{n_S^2 + 2n_O^2}{3} \qquad \text{(C.5)}$$

$$\Delta n = n_S - n_O \quad , \qquad \text{(C.6)}$$

where n_s and n_o are, respectively, the mean refractive index and the birefringence.

[1] $n = \sqrt{\varepsilon_0 \mu_0}$ from Maxwell equations.

Deriving with respect to the temperature, we obtain:

$$\frac{d}{dT} n_S = \frac{d}{dT}\langle n\rangle + \frac{2}{3}\frac{d}{dT}\Delta n \qquad \text{(C.7a)}$$

$$\frac{d}{dT} n_O = \frac{d}{dT}\langle n\rangle - \frac{1}{3}\frac{d}{dT}\Delta n \qquad \text{(C.7b)}$$

For nematic liquid crystals, the mean refractive index decreases linearly with temperature [6], so we can write:

$$\langle n\rangle = A - BT \qquad \text{(C.8)}$$

From the Haller semi-empirical equation [7]:

$$S = \left(1 - \frac{T}{T_C}\right)^{\beta} \qquad \text{(C.9)}$$

where S is a order parameter[2], T the temperature, T_C the transition temperature and β an exponential factor.

At this point, we can write the birefringence as a function of the order parameter:

$$\Delta n(T) = (\Delta n)_0 \cdot S \qquad \text{(C.10)}$$

Where $(\Delta n)_0$ is the birefringence at T = 0 K.

Replacing C.9 in C.10, we obtain:

$$\Delta n(T) = (\Delta n)_0 \cdot \left(1 - \frac{T}{T_C}\right)^{\beta} \qquad \text{(C.11)}$$

Equation C.11 gives the relation between the birefringence and the temperature. Replacing it and the equation C.8 in equation C.4a and C.4b, we obtain:

---

[2] $S = \left\langle \frac{3}{2}\cos^2\theta - \frac{1}{2}\right\rangle$ where θ stands for the angle between the main axes of a particular molecule and the average orientation of all molecules, and < > means an average over the entire system.

$$n_S = A - BT + \frac{2}{3}(\Delta n)_0 \cdot \left(1 - \frac{T}{T_C}\right)^{\beta} \qquad \text{(C.12a)}$$

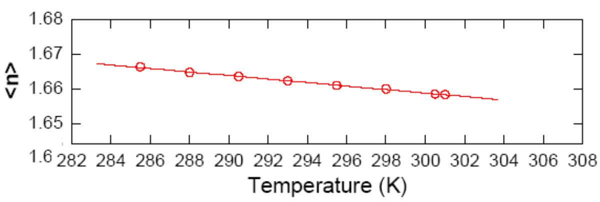
$$n_O = A - BT - \frac{1}{3}(\Delta n)_0 \cdot \left(1 - \frac{T}{T_C}\right)^{\beta} \qquad \text{(C.12b)}$$

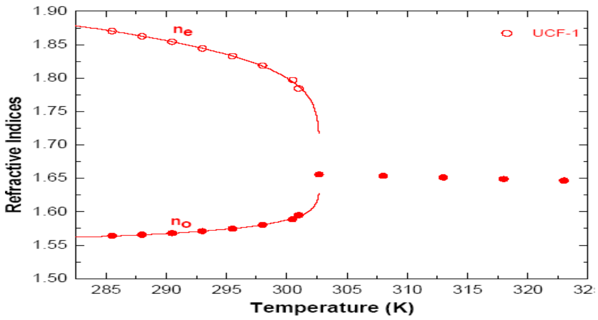
Deriving this equations respect to the temperature we obtain:

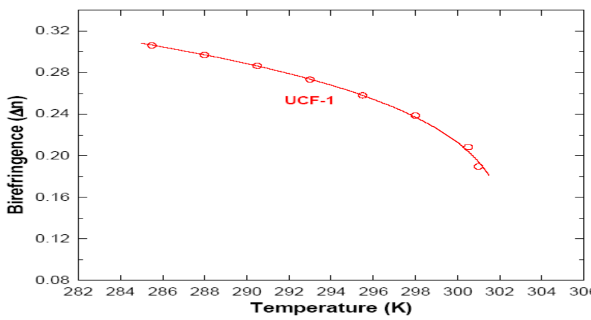
$$\frac{d}{dT} n_S = -B - \frac{2\beta(\Delta n)_0}{3T_C\left(1 - \frac{T}{T_C}\right)^{1-\beta}} \qquad \text{(C.13a)}$$

$$\frac{d}{dT} n_O = -B + \frac{\beta(\Delta n)_0}{3T_C\left(1 - \frac{T}{T_C}\right)^{1-\beta}} \qquad \text{(C.13b)}$$

From this equations, we observe that the n_S derivate with respect to the temperature consists of two negative terms, so we deduce that the refraction index decreases with the amount of the temperature in the whole nematic range. Contrary n_O has an inversion point where the derivate is zero. Here are reported some graphics extracts from [6].







Temperature variation influences also other physical parameters, like the dielectric constant, the elastic constant and the rotational viscosity. Here are reported the relations between these parameters and *S* [8].

$$\Delta n = (\Delta n)_0 \cdot S$$

$$\Delta \varepsilon = A \cdot \frac{S}{T}$$

where A is a proportionality factor

$$K_{ii} = (K_{ii})_0 \cdot S^2$$

where $(K_{ii})_0$ is the elastic constant at T = 0 K.

$$\gamma_1 = bSe^{E'/kT}$$

where γ_1 is the rotational viscosity, *b* is a proportionality factor, *E'* is the activation energy and *k* the Boltzmann constant.

Since the birefringence causes the polarimetric retardance, and makes the liquid crystal like a wave plate, we can apply the same relation of birefringence (C.10) to the retardance too:

$$\delta(T) = \delta_0 S = \delta_0 \left(1 - \frac{T}{T_C}\right)^{\beta} \qquad \text{(C.14)}$$

Where δ_0 is the retardance at T = 0 K.

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