Polarizers for Vacuum Ultraviolet. Simulation of performances for fluorides materials

Capobianco G. , Fineschi S.

Report nr. 122 date: 09/09/2009

INDEX

Abstract

Purpose of this Technical Report is to analyze the polarization performances of different materials at 121.6 nm wavelength. Materials used like linear polarizers at this wavelength usually are the fluorides: Magnesium Fluoride [$\mathbf{MgF_2}$], Lithium Fluoride [**LiF**] and Calcium Fluoride [$\mathbf{CaF_2}$]. Results are discussed and compared. For all materials, pile-of-plates configurations are also analyzed (transmitting linear polarizers).

1. Transmitting Linear Polarizer

A polarizer in Vacuum UV can be constructed by using a pile-of-plates of "transparent" material. The degree of polarization [p] is proportional to number of surfaces [m] by the law [Ref. 1]:

$$p = \frac{1-\left[\dfrac{2n^2}{1+n^4}\right]^m}{1+\left[\dfrac{2n^2}{1+n^4}\right]^m} \qquad \text{[eq. 1]}$$

Where n is the index of refraction of a chosen material at selected wavelength. This law is correct for incidence angle near to Brewster angle. Multiple refractions between plates are omitted (but not multiple refractions within plates).

Figure 1.1 shows that the degree of polarization at room temperature, and at 121.6 nm, for MgF_2 and LiF is higher than 95% for a number of surfaces greater or equal to 10. For CaF_2, 6 surfaces suffice.

Indices of refraction for these materials, at 121.6 nm wavelength, are [Ref 2-3-4]:

$$\begin{cases} n_{LiF} = 1.624 \\ n_{MgF_2} = 1.6275 \\ n_{CaF_2} = 2.02 \end{cases} \qquad \text{[eq. 2]}$$

These values are over plotted in Figure 1.1 using dashed blue line for LiF, dotted red line for MgF_2 and dotted green line for CaF_2.

MgF_2 is a birefringent material, and the index of refraction in eq. 2 is the ordinary refractive index [n_o]. The extraordinary refractive index at the same wavelength and temperature is: $n_{MgF_2}(extraordinary) = 1.6271$ [Ref 2].

The index of birefringence is: $\Delta n_{MgF_2} = n_o - n_e = 0.0004$

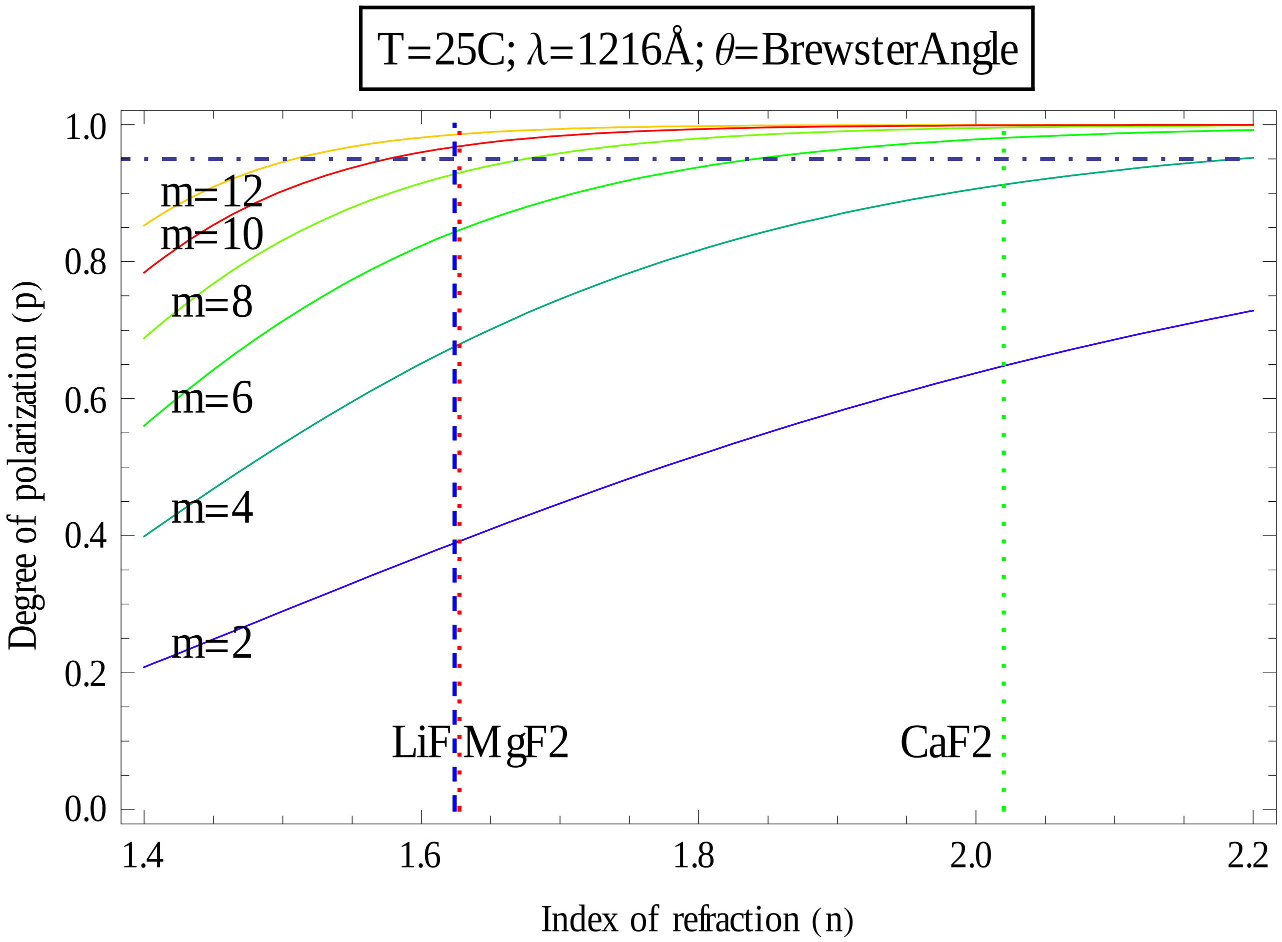

Figure 1.1 – *Degree of polarization (p) vs. Index of refraction (n) for 2,4,6,8,10 and 12 surfaces (m). Dashed blue line: index of refraction of LiF at 121.6 n;, dotted line: the ordinary index of refraction of* MgF_2 *at the same wavelength and dotted green line index of refraction of* CaF_2*.*

Assuming a 5-plate polarizer for LiF and MgF_2 (10 surfaces) and 3 plates (6 surfaces) for CaF_2, degree of polarizations are:

$$\begin{cases} p_{LiF} = 0.968 \\ p_{MgF_2} = 0.969 \\ p_{CaF_2} = 0.981 \end{cases} \qquad \text{[eq. 3]}$$

Incrementing number of plates, transmissivity decrease:

$$T = T_1^{m/4} \qquad (0 \le T_1 \le 1) \qquad \text{[eq. 4]}$$

Where T_1 is transmissivity for a single plate and m is the number of surfaces.

Figure 1.2 shows the degree of polarization (p), and transmissivity (T) as a function of the number of surfaces (m) for LiF, MgF_2 and CaF_2.

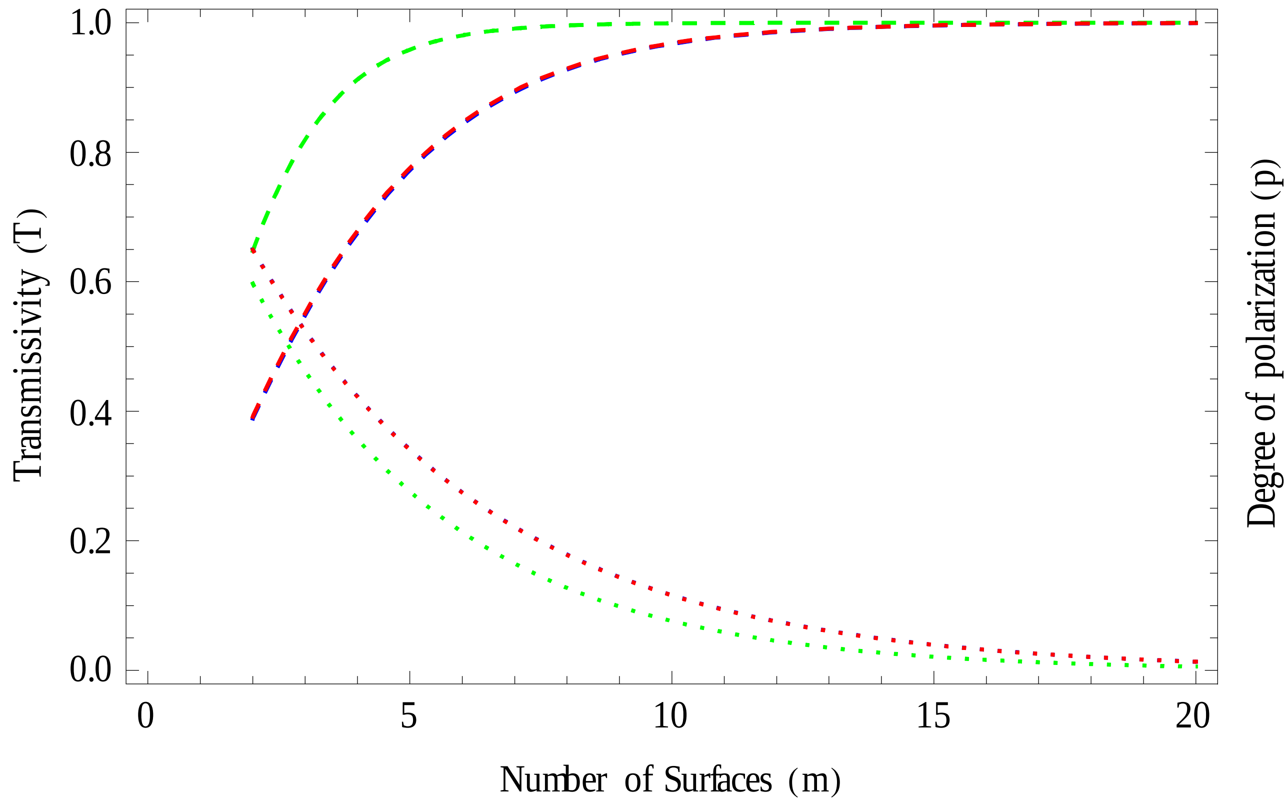

Figure 1.2 – *Degree of polarization (p) and Transmissivity (T) vs. Number of plates (m). Dashed lines are the degree of polarization of LiF (blue), MgF_2 (red) and CaF_2 (green). Dotted lines are transmissivity (T) of LiF (blue), MgF_2 (red) and CaF_2 (green).*

Using 5 plates for LiF and MgF2 and 3 plates for CaF_2, transmissivities are:

$$\begin{cases} T_{LiF} = 0.116 \quad (T_1 = 0.4227) \\ T_{MgF_2} = 0.116 \quad (T_1 = 0.4221) \\ T_{CaF_2} = 0.213 \quad (T_1 = 0.3570) \end{cases} \qquad \text{[eq. 5]}$$

As showed, LiF and MgF_2 have similar performances. CaF_2 have a better degree of polarization (higher index of refraction).

2. Reflecting Linear Polarizer

Reflecting linear polarizers are the most used in vacuum ultraviolet region. The index of ability of a given material to polarize by reflection is the *modulation factor* (μ), defined as:

$$\mu = \frac{R_s - R_p}{R_s + R_p} \qquad \text{[eq. 6]}$$

Where R_s is the reflectance of the component of electric vector perpendicular to the plane of incidence and R_p is the parallel component. R_s and R_p are evaluable by Fresnel and Snell laws:

$$R_s = \left[\frac{\cos(\vartheta_i) - \sqrt{\left({n_2}/{n_1} \right)^2 - \sin^2(\vartheta_i)}}{\cos(\vartheta_i) + \sqrt{\left({n_2}/{n_1} \right)^2 - \sin^2(\vartheta_i)}} \right]^2 \qquad \text{[eq. 7]}$$

$$R_p = \left[\frac{\left({n_2}/{n_1} \right)^2 \cos(\vartheta_i) - \sqrt{\left({n_2}/{n_1} \right)^2 - \sin^2(\vartheta_i)}}{\left({n_2}/{n_1} \right)^2 \cos(\vartheta_i) + \sqrt{\left({n_2}/{n_1} \right)^2 - \sin^2(\vartheta_i)}} \right]^2 \qquad \text{[eq. 8]}$$

Where both reflectances are in function only of the angle of incidence (ϑ_i) and index of refraction (n_2 and n_1). The average reflectivity is:

$$R = \frac{1}{2}\left(R_s + R_p\right) \qquad \text{[eq. 9]}$$

Transmissivity is:

$$T = 1 - R_s \qquad \text{[eq. 10]}$$

and normalized, so that the maximum value is 0.5.

A "*figure of merit*" that describes the non-ideality of polarizer is (Ref. Fineschi, 1999 SPIE):

$$\kappa = \mu\sqrt{R} \qquad \text{[eq. 11a]}$$

In transmission this "*figure of merit*" is:

$$\kappa = \mu\sqrt{T} \qquad \text{[eq. 11b]}$$

Where T is transmissivity normalized to 50%.

The Brewster angle is simply calculated as:

$$\vartheta_B = \arctan\left(\frac{n_2}{n_1} \right) \qquad \text{[eq. 12]}$$

Assuming the first medium vacuum:

$$\boldsymbol{n}_1 = 1. \qquad \text{[eq. 13]}$$

Indices of refraction for the analyzed materials are listed in eq. 2.

We assume that *extinction coefficient* is zero for all materials at 121.6 nm wavelength [Ref. 4].

In Figure 2.1, 2.2 and 2.3 are plotted this parameters vs. angle of incidence for LiF, MgF_2 and CaF_2. Dashed Red line is R_s, dashed blue line is R_p, dashed green line is R, solid black line is μ, dotted black line is κ, dotted orange line is normalized T and vertical dot-dashed line is Brewster angle.

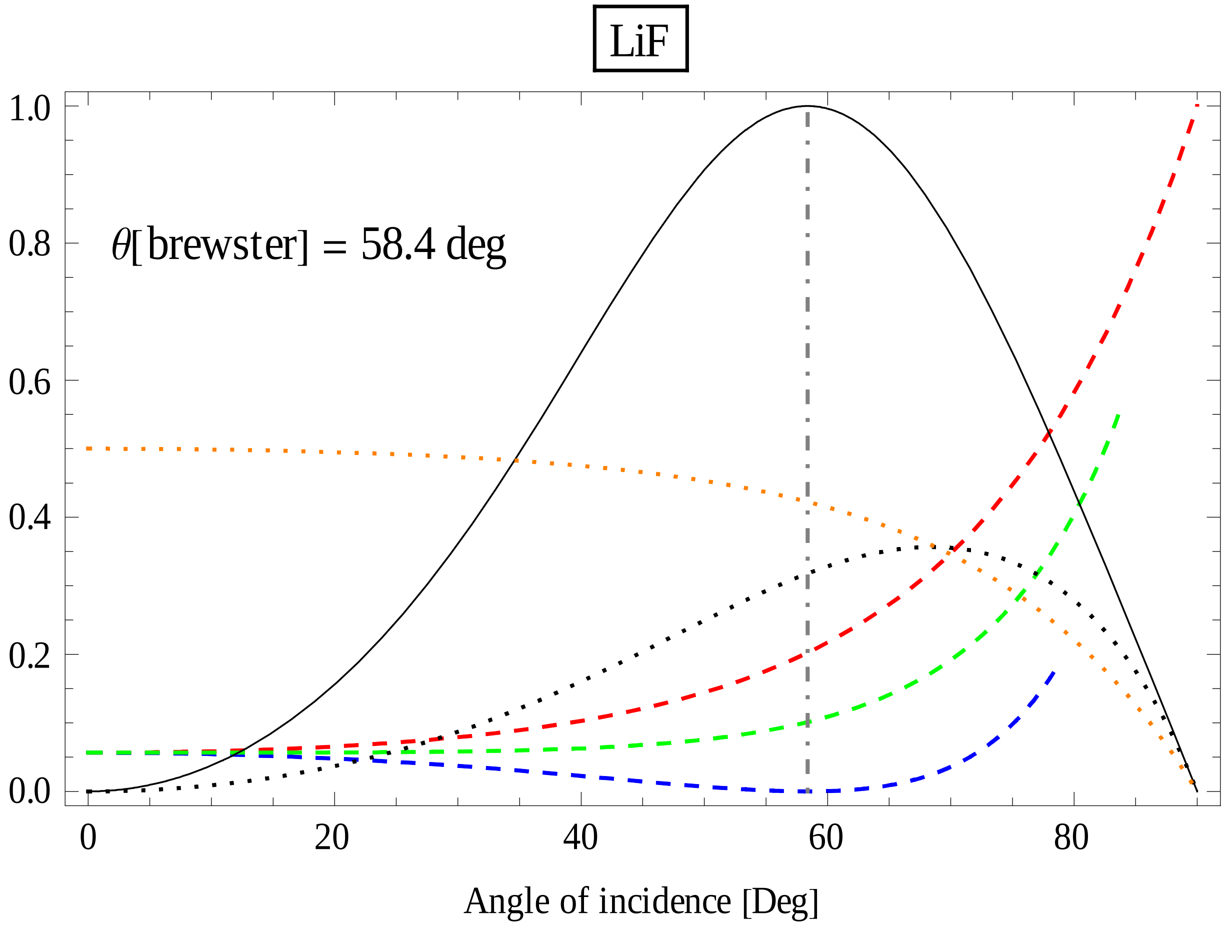

Figure 2.1 – R_s, R_p, R, μ, κ, T *vs. angle of incidence*

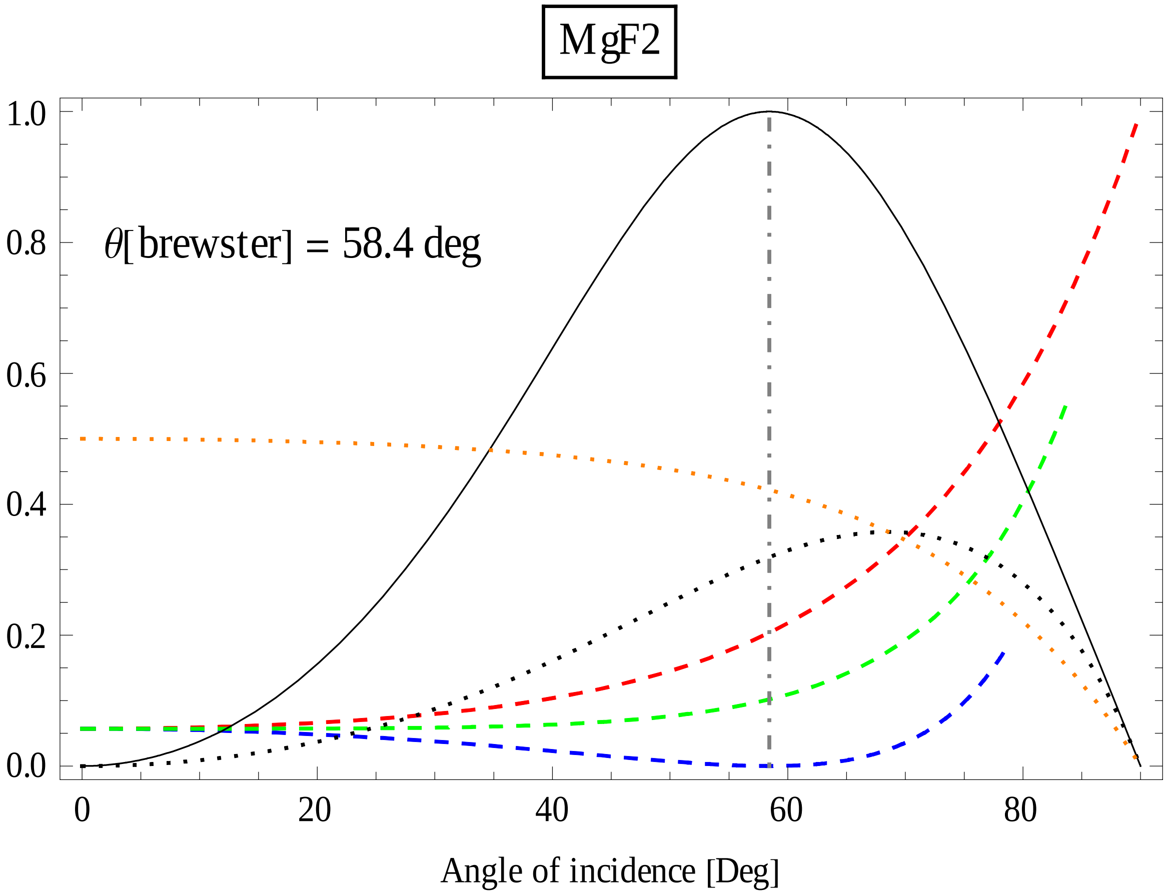

Figure 2.2 – *R_s, R_p, R, μ, κ, T vs. angle of incidence*

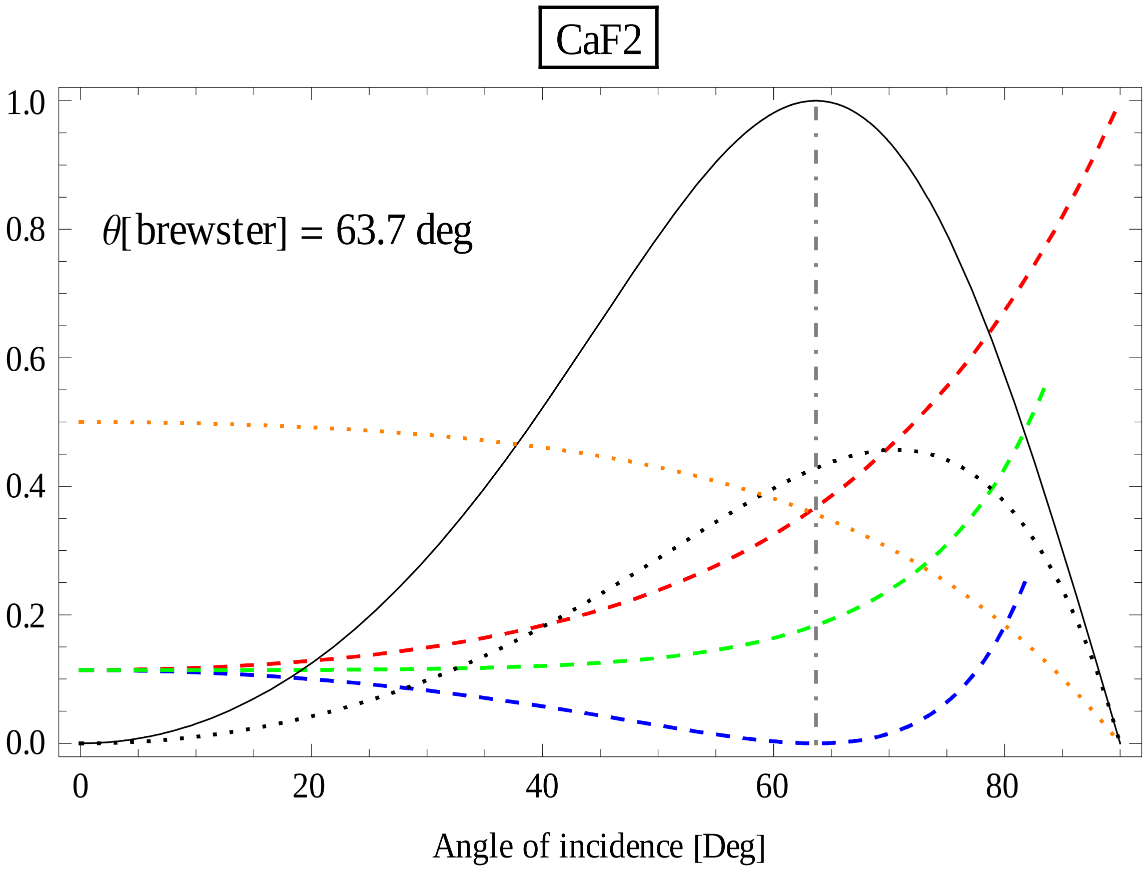

Figure 2.3 – *R_s, R_p, R, μ, κ, T vs. angle of incidence*

From eq. 11a and 11b, we evaluate the "figures of merit" for this three materials in transmission (using 5 plates of MgF_2 and LiF and 3 plates of CaF_2) and in reflection. The results are in Fig. 2.4.

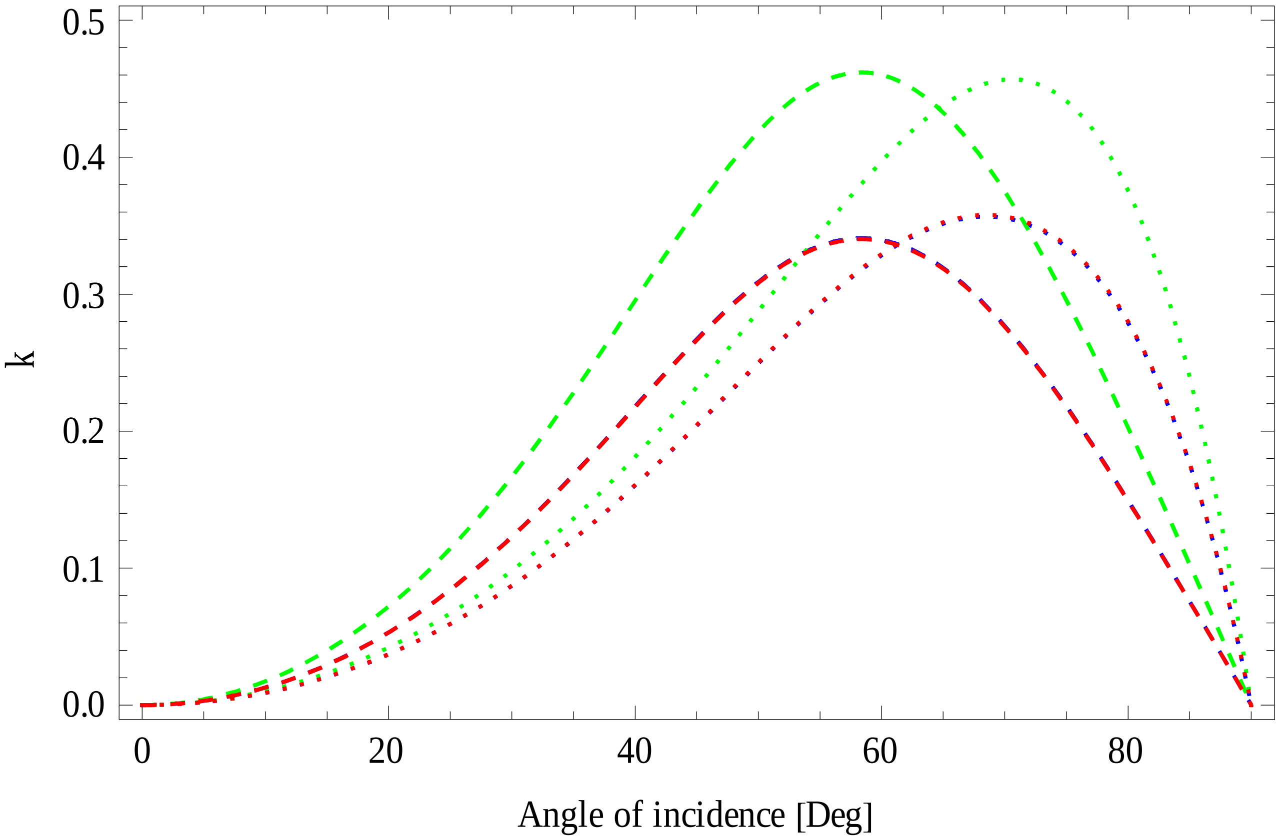

Figure 2.4 –*Figure of merit (k) vs. angle of incidence. Dashed lines are k in transmissionnfor LiF (blue), MgF_2 (red) and CaF_2 (green). Dotted lines are k in reflection for LiF (blue), MgF_2 (red) and CaF_2 (green).*

Assuming an even number of plates, than 6 plates for LiF and MgF_2 and 4 for CaF_2, the plot of "figure of merit" vs. angle of incidence is reported in Fig. 2.5. Transmissivity is reduced and at Brewster angle, *k* is higher in reflection case.

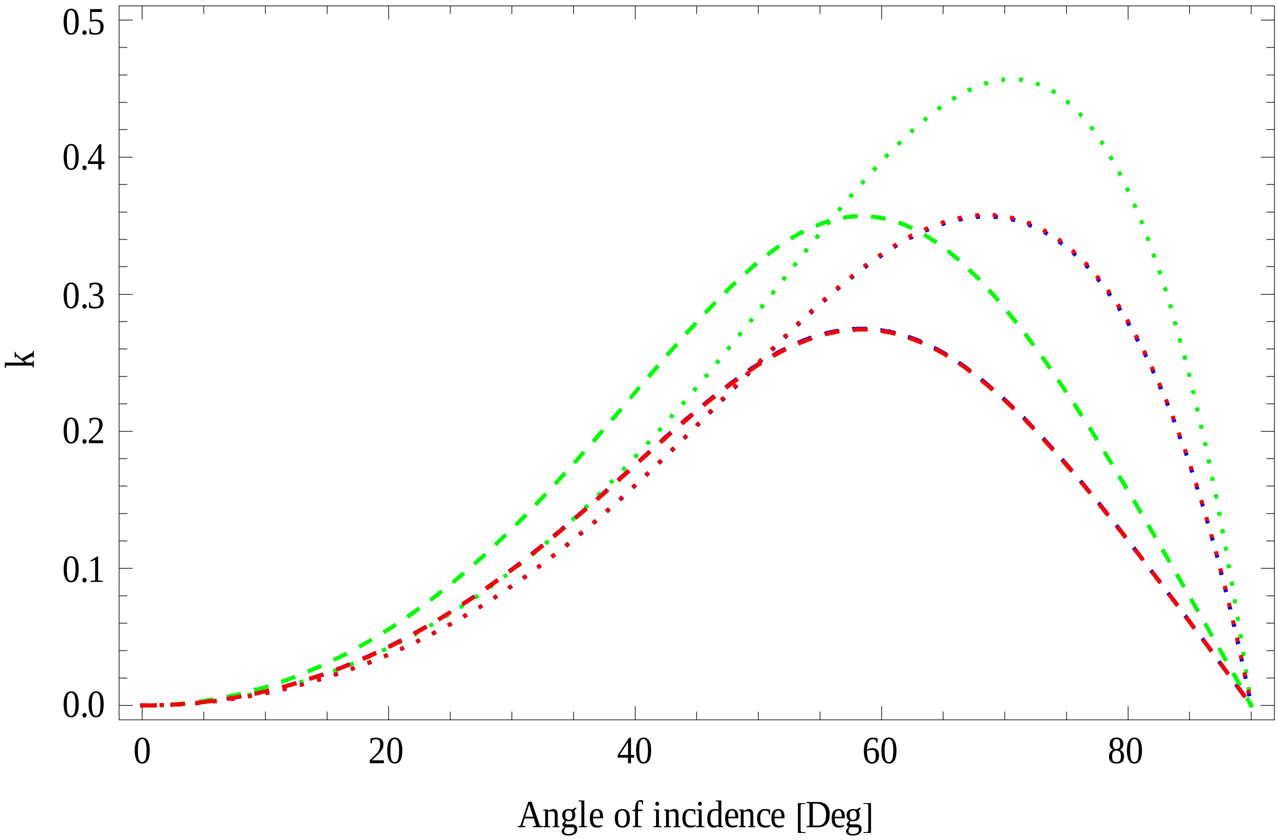

Figure 2.5 –*Figure of merit (k) vs. angle of incidence using even plates: 6 for LiF (blue), MgF_2 (red) and 4 for CaF_2 (green) . Dashed lines are k in transmission for LiF (blue), MgF_2 (red) and CaF_2 (green). Dotted lines are k in reflection for LiF (blue), MgF_2 (red) and CaF_2 (green).*

3. Conclusions

Summarizing:

- Magnesium Fluoride [**MgF_2**]: in transmission, has a degree of polarization up to 95% when using a medium/high number of plates (~ 5/6). In reflection, this material has similar performances. MgF_2 is birefringent and can introduce circular polarization (calibration may not be easy).
- Lithium Fluoride [**LiF**]: it has performances similar to those of MgF_2, but it is not birefringent. Indices of refraction have little variation in a sensible range of wavelengths near to HI Ly-α.
- Calcium Fluoride [**CaF_2**]: in transmission gives a good degree of polarization with few plates. Transmissivity is generally low. In reflection, it has good performances. It is also environmentally stable. ..

The trade-off between transmitting and reflecting linear polarizer is linked to a simpler, on-axis geometry for the transmitting versus better performances (i..e., higher figure of merit) for the reflecting (cfr. Fig.2.5).

References

[Ref 1] J.A. *Samson and D.L. Ederer* - Vacuum Ultraviolet Spectroscopy – Academic Press (2000)

[Ref 2] *Laporte et al.* - J.Opt. Soc. Am. Vol. **73**, No. 8 Pg. 1062-1069 (August 1983)

[Ref 3] Handbook of Optics Vol. II – McGraw-Hill (1995)

[Ref 4] *Fineschi et al.* – Proc. of SPIE Vol. 3764 Pg. 147-160 (1999).